

## The real primes and the Riemann hypothesis

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### Abstract

In this document, we deal with the concept of real prime number together with the Riemann hypothesis to the real numbers. Thus, we highlight the new hypothesis by a calculus of integral.

### The approach

The Riemann hypothesis states the non trivial zeros of the Riemann zeta function

$$\zeta(z) = \sum_{t=1}^{\infty} \frac{1}{t^z} \text{ lie on the critical line } \frac{1}{2} + iy$$

### Definition

A real number is composed if it can be written as  $\prod_j p_j^{n_j}$  where  $p_j$  are integer primes and  $n_j$  are rationals. A prime real number can be written only as  $p^{-1}$ . Thus, we define other real prime

numbers like  $\pi$ ,  $e$ ,  $\ln(2)$ . Thus  $\sqrt[q]{p} = p^{\frac{1}{q}}$  is composed. Also  $\sqrt[q]{p+1} = p^{\frac{1}{q}} + 1$  is prime and we have  $\sqrt[q]{p} - 1 = (p-1)(\sqrt[q]{p} + 1)^{-1}(\sqrt[q]{p} + 1)^{-1} \dots (\sqrt[q]{p} + 1)^{-1}$  composed, for example.

For  $t$  integer, Euler has proved that  $\zeta(z) = \sum_{t=1}^{\infty} \frac{1}{t^z} = \prod_{primes} \frac{1}{1-p^{-z}}$ , for  $t$  real

$$\prod_{primes} \frac{1}{1-p^{-z}} = \int_1^{\infty} \frac{dt}{t^z} = \frac{1}{1-z} \left[ \frac{t^{1-z}}{1-z} \right]_1^{\infty} \text{ For } z = \frac{1}{2} + iy \Rightarrow \zeta\left(\frac{1}{2} + iy\right) = 0$$

We have proved the hypothesis for real numbers. The Riemann hypothesis is important because it gives information about the zeros of the Riemann function and the distribution of those zeros are related to real primes!

### Conclusion

We did not present it like this, but we have given a proof of the Riemann hypothesis for real numbers.

### Bibliography

[1] Karl Sabbagh, The Riemann hypothesis, the greatest unsolved problem in mathematics, Farrar, Straus and Giroux, 2004.