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On Andrica's Conjecture, Cramér's Conjecture, gaps Between Primes and Jacobi Theta Functions II: A Simple Proof of Asymptotic for Andrica's Conjecture

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1. PRELIMINARES

In [1, p. 185] states that the prime number theorem yields:

$$p_n \sim n \log n, \tag{1}$$

that is,

$$\lim_{n\to\infty} \frac{p_n}{n \log n} = 1. \tag{2}$$

LEMMA 1. *The Andrica's conjecture is equivalent to*

$$p_{n+1} < 1 + 2\sqrt{p_n} + p_n. \tag{3}$$

Proof. Step 1. In [2, p.___], we conclude that Andrica's conjecture is equivalent to

$$\sqrt{p_n} < \frac{\theta_2}{\theta_3 - \theta_2} \Leftrightarrow \frac{1}{\theta_2} < [illegible] \frac{1}{\sqrt{p_n}}, \tag{4}$$

where $k := \frac{p_n}{p_{n+1}}$ is a k modulus. In [3, p. 83], we encounter

$$\frac{[illegible]}{\theta_3} = k^{1/2}. \tag{5}$$

Substituting (5) in (4) and considering $k = \frac{p_n}{p_{n+1}}$, we have

$$\sqrt{p_n} < \frac{\theta_2}{\theta_3 - \theta_2} \Leftrightarrow \frac{\sqrt{p_{n+1}}}{\sqrt{p_n}} < 1 + \frac{1}{\sqrt{p_n}}, \tag{6}$$

ergo, squaring both sides of the equation (6),

$$\frac{p_{n+1}}{p_n} < 1 + \frac{2}{\sqrt{p_n}} + \frac{1}{p_n} \Leftrightarrow p_{n+1} < 1 + 2\sqrt{p_n} + p_n. \square$$

2. THEOREM

THEOREM 1 (Asymptotic for Andrica's Conjecture). *Let $n \in \mathbb{N}$ and n sufficiently large, then*

$$\sqrt{p_{n+1}} - \sqrt{p_n} < 1.$$

Proof. Henceforth, we will use the *reductio ad absurdum* to prove the Lemma 1. We assume that

$$p_{n+1} \geq 1 + 2\sqrt{p_n} + p_n. \tag{7}$$

For n sufficiently large, we set (1) in (7), as follows

$$(n+1)\log(n+1) \geq 1 + 2\sqrt{n\log n} + n\log n. \tag{8}$$

Dividing (8) by $\log(n+1)$, we encounter

$$n+1 \geq \frac{1}{\log(n+1)} + 2\frac{\sqrt{n\log n}}{\log(n+1)} + n\frac{\log n}{\log(n+1)}. \tag{9}$$

On the other hand, it is easy to see that, as n is sufficiently large, then

$$\frac{\log n}{\log(n+1)} \longrightarrow 1, \qquad \frac{1}{\log(n+1)} \longrightarrow 0, \qquad \frac{\sqrt{n\log n}}{\log(n+1)} \longrightarrow \infty. \tag{10}$$

namely,

$$\lim_{n\to\infty}\frac{\log n}{\log(n+1)} = 1, \qquad \lim_{n\to\infty}\frac{1}{\log(n+1)} = 0, \qquad \lim_{n\to\infty}\frac{\sqrt{n\log n}}{\log(n+1)} = \infty.$$

Substituting (10) in (9), we find

$$n+1 \geq 0 + 2(\infty) + n\cdot 1 \Leftrightarrow 1 \geq 2(\infty), \tag{9}$$

which is false. Therefore, for n sufficiently large, $p_{n+1} < 1 + 2\sqrt{p_n} + p_n$. In face of Lemma 1, the asymptotic for Andrica's conjecture is proved. □

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