

AN UNORTHODOX PARAMETRIC MEASURE OF INFORMATION AND CORRESPONDING MEASURE OF INTUITIONISTIC FUZZY INFORMATION

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Abstract: A new parametric function

$$v_a(p) = \log(1 + a) - \sum_{i=1}^n \log\left(\frac{1 + ap_i}{p_i}\right) - 1, a > 0 \quad (2.1.1)$$

is proposed for the probability distribution $p_1, p_2, p_3, \dots, p_n$ and its properties are studied. In this paper the given functions is twice differentiable and is used to obtain the related measure of directed divergence, measure of intuitionistic fuzzy entropy, measure of intuitionistic fuzzy directed divergence. We also investigate the monotonic character of the proposed function.

1. Introduction

In the present paper we draw our inspiration for obtaining a new parametric measure of entropy which is the joint effect of measures of information due to Kapur [6] and Burg [2]. In 1948, C.E. Shannon [8] gave the measure

$$S(P) = - \sum_{i=1}^n p_i \log p_i \quad (1.1)$$

to measure its uncertainty or entropy. It can also be regarded as a measure of equality of $p_1, p_2, p_3, \dots, p_n$ among themselves.

Later in 1972, J.P. Burg [2] and J.N. Kapur [6] gave the measure

$$B(P) = \sum_{i=1}^n \log p_i \quad (1.2)$$

and

$$K_p(P) = - \sum_{i=1}^n p_i \log p_i + \frac{1}{a} \sum_{i=1}^n (1 + ap_i) - \frac{1}{a} (1 + a) \log(1 + a) \quad (1.3)$$

Shannon's and Burg's measures do not have any parameter, while Kapur's measure has one parameter. When maximized by Lagrange's method, subject to linear constraints on probabilities, the measures due to Shannon, Burg and Kapur always give non-negative probabilities. Shannon's measure has been the most successful and most widely used measure. Burg's measure has also been successful, but it is always negative and as such it is hard to interpret it as a measure of uncertainty. However, it can be used for entropy maximization purposes and it has been so used. Moreover, its maximum value decreases with n and this is not a desirable property for a measure of entropy. In the present discussion, we modify the Burg's and Kapur's measures to obtain a new parametric measure of entropy. We shall also study the properties of the measure and also investigate the related directed divergence motivated by Kullback-Liebler [7]. In this paper we introduce the

corresponding measure of intuitionistic fuzzy entropy and measure of intuitionistic fuzzy directed divergence.

Here we define intuitionistic fuzzy set as given by Atanasov (1983) [1] and then we discuss about the conditions of measure of intuitionistic fuzzy entropy.

Intuitionistic fuzzy set: - Let a set be fixed. An intuitionistic fuzzy set (IFs) A of E is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in E \}$ where the function $\mu_A: E \rightarrow [0,1]$ and $\nu_A: E \rightarrow [0,1]$ define respectively the degree of membership and degree of non membership of the element $x \in E$ to the set A , which is a subset of E and for every $x \in E, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$

Conditions for measures of intuitionistic fuzzy entropy:-

1. It should be defined in the range $0 \leq \mu_A(x) + \nu_A(x) \leq 1$
2. It should be continuous in this range.
3. It should be zero when $\mu_A(x) = 0$ and $\nu_A(x) = 1$.
4. It should not be changed when $\mu_A(x)$ changed in to $\nu_A(x)$.
5. It should be increasing function of $\mu_A(x)$ in the range $0 \leq \mu_A(x) \leq 0.5$ and decreasing function of $\nu_A(x)$ in the range $0 \leq \nu_A(x) \leq 0.5$
6. It should be concave function $\mu_A(x)$.

2. Our Results

2.1. Some Properties of the New Measures of Information

The measure is defined by

$$\nu_a(p) = \log(1 + a) - \sum_{i=1}^n \log\left(\frac{1 + ap_i}{p_i}\right) - 1, a > 0 \tag{2.1.1}$$

It has the following properties:

1. It is a continuous function of $p_1, p_2, p_3, \dots, \dots, p_n$ so that it changes by a small amount when $p_1, p_2, p_3, \dots, \dots, p_n$ change by small amounts.
2. It is a permutationally symmetric function of $p_1, p_2, p_3, \dots, \dots, p_n$ i.e. the function does not change when $p_1, p_2, p_3, \dots, \dots, p_n$ are permuted among themselves.
3. It is maximum, subject to natural constrai

$$\sum_{i=1}^n p_i = 1$$

When $p_1 = p_2 = \dots \dots \dots = p_n = \frac{1}{n}$ (2.1.2)

4. The maximum value is an increasing function of n . In fact the maximum value is given by

$$f(n) = \log(1 + a) - \sum_{i=1}^n \log(1 + a/n) + \sum_{i=1}^n \log \frac{1}{n} - 1 \tag{2.1.3}$$

$$= \log(1 + a) - n \log\left(\frac{n + a}{n}\right) + n \log \frac{1}{n} - 1 \tag{2.1.4}$$

so that $f'(n) = \frac{-n}{n+a} + \log\left(\frac{1}{n+a}\right)$ (2.1.5)

and $f''(n) = (-a)/(n + a)^2 - 1/(n + a) < 0$ (2.1.6)

So that $f(n)$ is a concave function of n and $f'(n)$ is a monotonic increasing function of n .

Now
$$f'(n) = \frac{-n}{n+a} + \log\left(\frac{1}{n+a}\right)$$

$$= \frac{a}{n+a} - 1 + \log\left(\frac{1}{n+a}\right) \quad (2.1.7)$$

$$= \frac{a}{y} - 1 + \log\left(\frac{1}{1}\right)$$

$$= \frac{a - y + \log\left(\frac{1}{y}\right)}{y}; y = n + a > 0 \quad (2.1.8)$$

Since $y > 0$ and $a - y + \log\left(\frac{1}{y}\right) < 0$ when $y > 0$ (2.1.9)

We get $f'(n) < 0$ (2.1.10)

So that $f(n)$ is monotonically increasing.

Thus, $v_a(p)$ satisfies all the important properties satisfied by Shannon's measure of entropy except additivity and recursivity. However, these properties are unimportant for entropy maximization purpose and hence, $v_a(p)$ is an entropy.

2.2. Corresponding Measure of Intuitionistic Fuzzy Information

$$V(A) = \log(1 + a) - \sum_{i=1}^n \log\left(\frac{1 + a\mu_A(x_i)}{\mu_A(x_i)}\right) - \sum_{i=1}^n \left(\frac{1 + av_A(x_i)}{v_A(x_i)}\right) - 1, a > 0 \quad (2.2.1)$$

It has the following properties:

1. It is defined in the range $0 \leq \mu_A(x_i) + v_A(x_i) \leq 1$
2. It is continuous in this range
3. It is zero when $\mu_A(x_i) = 0$ and $v_A(x_i) = 1$
4. It does not change when $\mu_A(x_i)$ changes into $v_A(x_i)$
5. It is an increasing function of $\mu_A(x_i)$ in the range $0 \leq \mu_A(x_i) \leq 0.5$ and a decreasing function of $v_A(x_i)$ in the range $0 \leq \mu_A(x_i) \leq 0.5$

Let $\mu_A(x_i) = s$ and $v_A(x_i) = t$ then

$$f(s, t) = \log(1 + a) - \log(1 + as) + \log s - \log(1 + at) + \log t - 1 \quad (2.2.2)$$

differentiate wrt. s we get

$$f'(s, t) = \frac{-1}{1 + as} + \frac{1}{s} \quad (2.2.3)$$

$$f'(s, t) = \frac{a}{(1 + as)^2} - \frac{1}{s^2} < 0 \quad (2.2.4)$$

So it is a concave function of $\mu_A(x_i)$

Since all the conditions for measures of intuitionistic fuzzy entropy are satisfied very well and therefore (2.2.1) is a valid measure of intuitionistic fuzzy entropy.

2.3. Corresponding Measure of Directed Divergence

Motivated by Kullback and Liebler [8] we get the corresponding measure of directed divergence,

$$D_a(P:Q) = \sum_{i=1}^n p_i \log \frac{p_i}{q_i} - \sum_{i=1}^n p_i \log \frac{1 + ap_i}{1 + aq_i} \quad (2.3.1)$$

Here $D_a(P:Q)$ satisfies the properties $D_a(P:Q) \geq 0$, vanishes iff $Q=P$ and is a convex function of both $p_1, p_2, p_3, \dots, p_n$ and $q_1, q_2, q_3, \dots, q_n$

2.4. Corresponding Measure of Intuitionistic Fuzzy Directed Divergence

$$D_a(A: B) = \sum_{i=1}^n \mu_A(x_i) \log \frac{\mu_A(x_i)}{\mu_B(x_i)} + \sum_{i=1}^n v_A(x_i) \log \frac{v_A(x_i)}{v_B(x_i)} - \sum_{i=1}^n \mu_A(x_i) \log \frac{1 + a\mu_A(x_i)}{1 + a\mu_B(x_i)} - \sum_{i=1}^n v_A(x_i) \log \frac{1 + av_A(x_i)}{1 + av_B(x_i)} \quad (2.4.1)$$

It is a convex function of $\mu_A(x_i)$. Since $D_a(A: B)$ satisfies all the conditions for measures of intuitionistic fuzzy directed divergence therefore (2.4.1) is a valid measure of intuitionistic fuzzy directed divergence.

3. References

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