Soliton Solutions of Space-Time Fractional-Order Modified Extended Zakharov-Kuznetsov Equation in Plasma Physics

Muhammad Nasir Ali^{1,a}, Syed Muhammad Husnine^{1,b}, Sana Noor^{2,c}
 and Turgut Ak^{3,d,*}

¹Department of Sciences and Humanities, National University of Computer and Emerging Sciences, Lahore, Pakistan

²Department of Geography, Lahore Collage for Women University, Lahore, Pakistan

³Armutlu Vocational School, Yalova University, Armutlu 77500, Yalova, Turkey

^ataj467@gmail.com, ^bsyed.husnine@nu.edu.pk, ^csananoor86@gmail.com, ^{d,*}akturgut@yahoo.com

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Abstract. The aim of this article is to calculate the soliton solutions of space-time fractional-order modified extended Zakharov-Kuznetsov equation which is modeled to investigate the waves in magnetized plasma physics. Fractional derivatives in the form of modified Riemann-Liouville derivatives are used. Complex fractional transformation is applied to convert the original nonlinear partial differential equation into another nonlinear ordinary differential equation. Then, soliton solutions are obtained by using $(1/G')$ –expansion method. Bright and dark soliton solutions are also obtained with ansatz method. These solutions may be of significant importance in plasma physics where this equation is modeled for some special physical phenomenon.

Introduction

Many real world problems in science and engineering are modelled by using nonlinear partial differential equations (NPDEs). Finding the exact solutions of such nonlinear equations is an important area of research. Fractional differential equations (FDE's) are also getting the attention of the researchers in the recent years. Many real world problems are modelled via FDE's in fluid dynamics. Exact solutions of such models play an important role in the mathematical sciences [1-9].

FDE's are studied by researchers for obtaining the exact solutions with different techniques in literature. Exact solutions of time-fractional Burgers equation, biological population model and space-time fractional Whitham-Broer-Kaup equations are calculated with (G'/G) –expansion method [10]. Lie group analysis for n order linear fractional partial differential equation and nonlinear fractional reaction diffusion convection equation are performed [11]. Homotopy perturbation method is applied to the nonlinear fractional Kolmogorov-Petrovskii-Piskunov equations to obtain exact solutions [12]. Exact solutions of (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation are obtained [13]. The improved extended tanh-coth method and $(1/G')$ –expansion method are used to find the exact solution of Sharma-Tasso-Olver equation [14,15]. New extended trial equation method is used for finding the exact solution of a class of generalized fractional Zakharov-Kuznetsov equation [16]. Improved fractional sub-equation method is utilized for calculating the exact solutions of (3+1)-dimensional generalized fractional KdV-Zakharov-Kuznetsov equations [17]. Generalized Kudryashov method is considered for time-fractional differential equations [18].

In mathematical physics, Zakharov-Kuznetsov (ZK) equation is used to describe the nonlinear development of ion-acoustic waves in magnetized plasma [19]. It is comprised of cold ions and hot isothermal electrons in the presence of a uniform magnetic field. It is also known as the generalization of KdV equation. Extended form of (1+2)-dimensional and (1+3)-dimensional quantum Zakharov-Kuznetsov equation are investigated for exact solutions [20].

In this article, we consider the following (1+3)-dimensional space-time fractional-order modified extended Zakharov-Kuznetsov equation (MEZK)

$$D_t^\alpha u + \beta u^2 D_z^\alpha u + \gamma (D_x^{3\alpha} u + D_y^{3\alpha} u + D_z^{3\alpha} u) + \delta (D_z^\alpha D_x^{2\alpha} u + D_z^\alpha D_y^{2\alpha} u) = 0, \quad (1)$$

for calculating the soliton solutions where β, γ, δ are the constants and $0 < \alpha \leq 1$.

The article is arranged as follows. In section, modified Riemann-Liouville derivative of order α is explained with some of its properties. Soliton solutions are obtained with $(1/G')$ –expansion method. Also, bright and dark soliton solutions are calculated with ansatz method in Section 3. Conclusion is provided in Section 4. References are given in the end.

Modified Riemann-Liouville Derivative and Its Properties

The modified Riemann-Liouville derivative of order α for a continuous function is defined as follows [21]

$$D_x^\alpha g(x) = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_0^x (x-\tau)^{-\alpha} (g(\tau) - g(0)) d\tau, & 0 < \alpha < 1 \\ (g^n(x))^{\alpha-n}, & n \leq \alpha < n+1, \quad n \geq 1 \end{cases} \quad (2)$$

where $g: R \rightarrow R$, $x \rightarrow g(x)$ denotes a continuous function not necessarily first order differential. Following are the important properties of modified Riemann-Liouville derivative.

- If $h: R \rightarrow R$, is a continuous function, then its fractional derivative in the form of integral w.r.t. $(dx)^\alpha$

$$D_x^\alpha g(x) = \frac{1}{\Gamma(\alpha)} \int_0^{x_1} (g - \tau)^{\alpha-1} g(\tau) d\tau = \frac{1}{\Gamma(1+\alpha)} \frac{d}{dx} \int_0^x g(\tau) (d\tau)^\alpha, \quad 0 < \alpha < 1 \quad (3)$$

- For any constant k , the fractional derivative is

$$D_x^\alpha(k) = 0. \quad (4)$$

- Fractional derivative for the linear combination of the functions $g(x)$ and $h(x)$ and the constants a and b is defined as

$$D_x^\alpha(a g(x) + b(h(x))) = a D_x^\alpha(g(x)) + b D_x^\alpha(h(x)). \quad (5)$$

- For $h(x) = (x)^p$, the fractional derivative will be

$$D_x^\alpha((x)^p) = \frac{\Gamma(1+p)}{\Gamma(1+p-\alpha)} (x)^{p-\alpha}. \quad (6)$$

Soliton Solutions of Fractional Form of MEZK Equation

In this section, different methods are applied to find the soliton solutions of Eq. (1).

$(1/G')$ –Expansion method

Here, we use the $(1/G')$ –expansion method for calculating the soliton solutions. For transforming the Eq. (1) in to another ordinary differential equation (ODE), we apply the following complex fractional transformation

$$\xi = \frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)}. \quad (7)$$

This results in the following ODE

$$-wU' + c\beta U^2 U' + (\gamma(a^3 + b^3 + c^3) + \delta(a^2 c + b^2 c))U''' = 0, \quad (8)$$

where $U = u(\xi)$ and $U' = \frac{du}{d\xi}$. Integrating Eq. (8) w.r.t. ξ taking constant of integration zero, it yields

$$-wU + \frac{1}{3}AU^3 + BU'' = 0. \quad (9)$$

where $A = c\beta$ and $B = \gamma(a^3 + b^3 + c^3) + \delta(a^2 c + b^2 c)$. Balancing the terms U'' and U^3 , we obtain $M = 1$. Hence the solution will take the form as

$$U = a_0 + a_1 \left(\frac{1}{G'} \right), \quad (10)$$

with a_0 and a_1 as constants to be determined and

$$\left(\frac{1}{G'} \right) = \frac{\lambda}{-\mu + \lambda c_1 (\cosh(\lambda \xi) - \sinh(\lambda \xi))}. \quad (11)$$

Also $G(\xi)$ will satisfy the ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu = 0$ where λ and μ are the constants. This equation contains the solution

$$G(\xi) = c_1 e^{-\lambda \xi} - \frac{\mu}{\lambda} \xi + c_2. \quad (12)$$

Using Eq. (10) in Eq. (9) and then comparing coefficients of different powers of $\left(\frac{1}{G'} \right)$, we obtain the following system of equations

$$\begin{aligned} \left(\frac{1}{G'} \right)^0: & \quad -wa_0 + \frac{A}{3}a_0^3 = 0, \\ \left(\frac{1}{G'} \right)^1: & \quad a_0^2 a_1 A + a_1 B \lambda^2 - wa_1 = 0, \\ \left(\frac{1}{G'} \right)^2: & \quad a_1^2 a_0 A + 3\mu \lambda a_1 B = 0, \\ \left(\frac{1}{G'} \right)^3: & \quad \frac{A}{3}a_1^3 + 2B\mu^2 a_1 = 0, \end{aligned} \quad (13)$$

Solving the system (13), we get the following solutions

$$\text{Set 1:} \quad \lambda^{(1)} = -i\sqrt{\frac{2w}{B}}, \quad a_1^{(1)} = -i\sqrt{\frac{6B}{A}}\mu, \quad a_0^{(1)} = -\sqrt{\frac{3w}{A}}. \quad (14)$$

$$\text{Set 2:} \quad \lambda^{(2)} = -i\sqrt{\frac{2w}{B}}, \quad a_1^{(2)} = i\sqrt{\frac{6B}{A}}\mu, \quad a_0^{(2)} = \sqrt{\frac{3w}{A}}. \quad (15)$$

$$\text{Set 3:} \quad \lambda^{(3)} = i\sqrt{\frac{2w}{B}}, \quad a_1^{(3)} = -i\sqrt{\frac{6B}{A}}\mu, \quad a_0^{(3)} = \sqrt{\frac{3w}{A}}. \quad (16)$$

$$\text{Set 4:} \quad \lambda^{(4)} = i\sqrt{\frac{2w}{B}}, \quad a_1^{(4)} = i\sqrt{\frac{6B}{A}}\mu, \quad a_0^{(4)} = -\sqrt{\frac{3w}{A}}. \quad (17)$$

Putting values from Eqs. (14-17) in Eq. (10) using Eq. (11), following

$$U_1 = -\sqrt{\frac{3w}{A}} - \left(i\sqrt{\frac{6B}{A}}\mu \right) \frac{\lambda^{(1)}}{-\mu + \lambda^{(1)} c_1 [\cosh(\lambda^{(1)} \xi) - \sinh(\lambda^{(1)} \xi)]}, \quad (18)$$

$$U_2 = \sqrt{\frac{3w}{A}} + \left(i \sqrt{\frac{6B}{A}} \mu \right) \frac{\lambda^{(2)}}{-\mu + \lambda^{(2)} c_1 [\cosh(\lambda^{(2)} \xi) - \sinh(\lambda^{(2)} \xi)]}, \quad (19)$$

$$U_3 = \sqrt{\frac{3w}{A}} - \left(i \sqrt{\frac{6B}{A}} \mu \right) \frac{\lambda^{(3)}}{-\mu + \lambda^{(3)} c_1 [\cosh(\lambda^{(3)} \xi) - \sinh(\lambda^{(3)} \xi)]}, \quad (20)$$

$$U_4 = -\sqrt{\frac{3w}{A}} + \left(i \sqrt{\frac{6B}{A}} \mu \right) \frac{\lambda^{(4)}}{-\mu + \lambda^{(4)} c_1 [\cosh(\lambda^{(4)} \xi) - \sinh(\lambda^{(4)} \xi)]}, \quad (21)$$

where ξ is defined in Eq. (7).

Ansatz method

To obtain bright and dark soliton solutions of Eq. (1), ansatz method is applied.

Bright soliton solutions

To obtain the bright solutions of Eq. (9), we consider the ansatz of the form

$$U = u(\xi) = \frac{\tau}{\cosh^p(\theta \xi)}, \quad (22)$$

where τ is the amplitude of the soliton, θ is the inverse width of the soliton and $p > 0$ for the solitons to exist. The value of the unknown p will be determined during the derivation of the solution. Now

$$U^3 = \frac{\tau^3}{\cosh^{3p}(\theta \xi)}, \quad (23)$$

$$U'' = \frac{\tau p^2 \theta^2}{\cosh^p(\theta \xi)} - \frac{\tau p(p+1)\theta^2}{\cosh^{p+2}(\theta \xi)}.$$

Putting the values in Eq. (9), it results

$$\frac{-w\tau}{\cosh^p(\theta \xi)} + \frac{A}{3} \frac{\tau^3}{\cosh^{3p}(\theta \xi)} + B \left(\frac{\tau p^2 \theta^2}{\cosh^p(\theta \xi)} - \frac{\tau p(p+1)\theta^2}{\cosh^{p+2}(\theta \xi)} \right) = 0. \quad (24)$$

A and B are defined above. Equating the exponents $3p$ and $p+2$ from Eq. (24), we have

$$3p = p + 2, \quad (25)$$

$$p = 1.$$

Now comparing the different powers of $\frac{1}{\cosh(\theta \xi)}$, it yields the following system

$$\frac{1}{3} A \tau^3 - 2B \tau \theta^2 = 0, \quad (26)$$

$$-w\tau + B \tau \theta^2 = 0.$$

Solving this system, one can obtain

$$\text{Set 1:} \quad \tau^{(1)} = -\sqrt{\frac{6w}{A}}, \quad \theta^{(1)} = -\sqrt{\frac{w}{B}}, \quad (27)$$

$$\text{Set 2: } \tau^{(2)} = -\sqrt{\frac{6w}{A}}, \quad \theta^{(2)} = \sqrt{\frac{w}{B}}, \quad (28)$$

$$\text{Set 3: } \tau^{(3)} = \sqrt{\frac{6w}{A}}, \quad \theta^{(3)} = -\sqrt{\frac{w}{B}}, \quad (29)$$

$$\text{Set 4: } \tau^{(4)} = \sqrt{\frac{6w}{A}}, \quad \theta^{(4)} = \sqrt{\frac{w}{B}}, \quad (30)$$

Hence the bright solutions will be

$$U_5 = \frac{-\sqrt{\frac{6w}{A}}}{\cosh \left[-\sqrt{\frac{w}{B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right]}, \quad (31)$$

$$U_6 = \frac{-\sqrt{\frac{6w}{A}}}{\cosh \left[\sqrt{\frac{w}{B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right]}, \quad (32)$$

$$U_7 = \frac{\sqrt{\frac{6w}{A}}}{\cosh \left[-\sqrt{\frac{w}{B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right]}, \quad (33)$$

$$U_8 = \frac{\sqrt{\frac{6w}{A}}}{\cosh \left[\sqrt{\frac{w}{B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right]}. \quad (34)$$

where ξ is defined in Eq. (7).

Dark soliton solutions

To calculate the dark solutions of Eq. (9), we take the ansatz of the form

$$U = u(\xi) = \tau \tanh^p(\theta \xi), \quad (35)$$

where $p > 0$ is unknown and will be determined during the derivation of the solution. Now

$$\begin{aligned} U^3 &= \tau^3 \tanh^{3p}(\theta \xi), \\ U'' &= \tau p \theta^2 ((p-1) \tanh^{p-2}(\theta \xi) - 2p \tanh^p(\theta \xi) + (p+1) \tanh^{p+2}(\theta \xi)). \end{aligned} \quad (36)$$

Putting the values in Eq. (9), it results

$$\begin{aligned} B \left(\tau p \theta^2 ((p-1) \tanh^{p-2}(\theta \xi) - 2p \tanh^p(\theta \xi) + (p+1) \tanh^{p+2}(\theta \xi)) \right) - w \tau \tanh^p(\theta \xi) \\ + \frac{A}{3} \tau^3 \tanh^{3p}(\theta \xi) = 0. \end{aligned} \quad (37)$$

A and B are defined above. Equating the exponents $3p$ and $p+2$ from Eq. (24), we have

$$\begin{aligned} 3p &= p+2, \\ p &= 1. \end{aligned} \quad (38)$$

Now equating the different powers of $\tanh(\theta \xi)$ yields the following system

$$\begin{aligned}\frac{1}{3}A\tau^3 + 2B\tau\theta^2 &= 0, \\ -w\tau - 2B\tau\theta^2 &= 0.\end{aligned}\tag{39}$$

Solving this system, one can obtain

$$\text{Set 1: } \tau^{(5)} = -\sqrt{\frac{3w}{A}}, \quad \theta^{(5)} = -i\sqrt{\frac{w}{2B}},\tag{40}$$

$$\text{Set 2: } \tau^{(6)} = -\sqrt{\frac{3w}{A}}, \quad \theta^{(6)} = i\sqrt{\frac{w}{B}},\tag{41}$$

$$\text{Set 3: } \tau^{(7)} = \sqrt{\frac{6w}{A}}, \quad \theta^{(7)} = -i\sqrt{\frac{w}{B}},\tag{42}$$

$$\text{Set 4: } \tau^{(8)} = \sqrt{\frac{6w}{A}}, \quad \theta^{(8)} = i\sqrt{\frac{w}{B}},\tag{43}$$

Hence the dark soliton solutions will be

$$U_9 = -\sqrt{\frac{3w}{A}} \tanh \left[-i\sqrt{\frac{w}{2B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right],\tag{44}$$

$$U_{10} = -\sqrt{\frac{3w}{A}} \tanh \left[i\sqrt{\frac{w}{2B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right],\tag{45}$$

$$U_{11} = \sqrt{\frac{3w}{A}} \tanh \left[-i\sqrt{\frac{w}{2B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right],\tag{46}$$

$$U_{12} = \sqrt{\frac{3w}{A}} \tanh \left[i\sqrt{\frac{w}{2B}} \left(\frac{a(x)^\alpha}{\Gamma(1+\alpha)} + \frac{b(y)^\alpha}{\Gamma(1+\alpha)} + \frac{c(z)^\alpha}{\Gamma(1+\alpha)} - \frac{w(t)^\alpha}{\Gamma(1+\alpha)} \right) \right].\tag{47}$$

where ξ is defined in Eq. (7).

Conclusion

In this article, space-time fractional form of MEZK is investigated for soliton solutions. Complex fractional transformation is utilized to achieve the nonlinear ODE from fractional MEZK equation. Bright and dark soliton solutions are obtained with solitary wave ansatz method. $(1/G')$ –expansion method is also applied to get some other solutions. These solutions may be of significant importance for the explanation of the some special physical phenomena arising in plasma physics modelled by this equation. This article also highlights the strength of the methods to obtain the soliton solutions to the highly nonlinear FDE's with constants coefficients.

Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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