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Computing F-Index of Different Corona Products of Graphs

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Abstract. F-index of a graph is equal to the sum of cubes of degree of all the vertices of a given graph. Among different products of graphs, as corona product of two graphs is one of most important, in this study, the explicit expressions for F-index of different types of corona product of are obtained.

Introduction

A topological index is defined as a real valued function, which maps each molecular graph to a real number and is necessarily invariant under automorphism of graphs. There are various topological indices having strong correlation with the physicochemical characteristics and have been found to be useful in isomer discrimination, quantitative structure-activity relationship (QSAR) and structure-property relationship (QSPR).

In this article, as a molecular graph, we consider only finite, connected and undirected graphs without any self-loops or multiple edges. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$ so that the order and size of G is equal to n and m respectively. Let the edge connecting the vertices u and v is denoted by uv. Let, the degree of the vertex v in G is denoted by $d_G(v)$, which is the number of edges incident to v, that is, the number of first neighbors of v.

Among various degree-based topological indices, the first ($M_1(G)$) and the second ($M_2(G)$) Zagreb index of a G are one of the oldest and most studied topological indices introduced in [13] by Gutman and Trinajstić and defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(u)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

and

$$M_2(G) = \sum_{u,v \in V(G)} d_G(u) d_G(v).$$

These indices have extensively studied both with respect to mathematical and chemical point of view. Other than first and second Zagreb indices another topological indices was introduced in [13] and is defined as

$$F(G) = \sum_{v \in V(G)} d_G(u)^3 = \sum_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2\right]$$

Furtula and Gutman in 2015 [12] studied this index again and named as ”forgotten topological index” or ”F-index”. In that paper they showed that the predictive ability of this index is almost similar to that of first Zagreb index and for the entropy and acetic factor, both of them yield correlation coefficients greater than 0.95. There are various recent mathematical as well as chemical study of F-index (for details see [4, 5, 6, 7, 1, 8]). Throughout this paper, as usual, C_n and P_n denote the cycle and path graphs on n vertices.

Among the most well known products of graphs, the corona product of graphs is one of the most important graph operations as different important class of graphs can be obtained by corona product of some general and particular graphs. For example, Corona product of graphs appears in chemical literature as plerographs of the hydrogen suppressed molecular graphs known as kenographs. Also, by specializing the components of corona product of graphs different interesting classes of graphs such as t-thorny graph, sunlet graph, bottleneck graph, suspension of graphs and some classes of bridge graphs can be obtained (for details see [3, 9, 10, 2, 17, 11]. In this paper, we derive some explicit expressions of different type of corona product of graphs such as subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood corona, subdivision-edge neighborhood corona and vertex-edge corona of two graphs.

Main Results

Let G_1 and G_2 be two simple connected graphs with n_i number of vertices and m_i number of edges respectively, for $i \in \{1, 2\}$. Different topological indices under the corona product of graphs have already been studied by some researchers. The corona product of $G_1 \circ G_2$ of these two graphs is obtained by taking one copy of G_1 and n_1 copies of G_2; and by joining each vertex of the i-th copy of G_2 to the i-th vertex of G_1, where $1 \leq i \leq n_1$. The corona product of G_1 and G_2 has total number of $(n_1n_2+n_1)$ vertices and $(m_1+n_1m_2+n_1n_2)$ edges. Note that corona product operation of two graphs is not commutative and in case of corona product of graphs we can obtain connected graphs having pendent vertices. Inspired by this original corona product of graphs, several authors defined other different versions of corona product of graphs such as subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood corona, subdivision-edge neighborhood corona and vertex-edge corona etc. and investigated them (see [14, 15, 16]). In this section, we proceed to introduce different type of corona product of graphs and hence find the explicit expressions of F-index of that corona product of two graphs. Recall that, the subdivision graph $S = S(G)$ is the graph obtained from G by replacing each of its edges by a path of length two, or equivalently, by inserting an additional vertex into each edge of G.

Subdivision-vertex corona

Definition 1. [14] Let G_1 and G_2 be two vertex disjoint graphs. The subdivision-vertex corona of G_1 and G_2 is denoted by $G_1 \odot G_2$ and obtained from $S(G_1)$ and n_1 copies of G_2, all vertex-disjoint, by joining the i-th vertex of $V(G_1)$ to every vertex in the i-th copy of G_2.

From definition it is clear that the subdivision-vertex corona $G_1 \odot G_2$ has $n_1(1+n_2)+m_1$ vertices and $2m_1 + n_1(n_2 + m_2)$ edges. Also, the degree of the vertices of $G_1 \odot G_2$ are given by

$d_{G_1 \odot G_2}(v_i) = d_{G_1}(v_i) + n_2$ for $i = 1, 2, ..., n_1$

$d_{G_1 \odot G_2}(e_i) = 2$ for $i = 1, 2, ..., m_1$

$d_{G_1 \odot G_2}(v_j^i) = d_{G_2}(u_j) + 1$ for $i = 1, 2, ..., n_1$and $j = 1, 2, ..., n_2$.

The subdivision-vertex corona of P_4 and P_2 is given in Figure 1. In the following theorem we calculate the F-index of the subdivision-vertex corona $G_1 \odot G_2$.

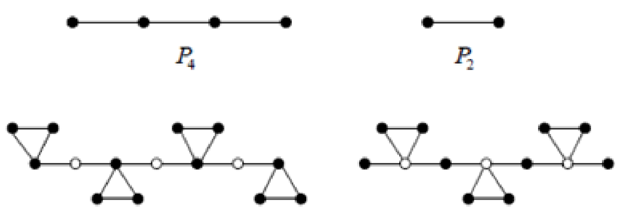

Figure 1: Subdivision-vertex and subdivision-edge corona products of P_4 and P_2.

Theorem 2. *The F-index of the subdivision-vertex corona $G_1 \odot G_2$ is given by*
$F(G_1 \odot G_2) = F(G_1) + n_1F(G_2) + 3n_2M_1(G_1) + 3n_1M_1(G_2) + 6{n_2}^2m_1 + n_1{n_2}^3 + 6n_1m_2 + n_1n_2 + 8m_1$.

Proof. From definition of subdivision-vertex corona $G_1 \odot G_2$, we get

$$\begin{aligned}F(G_1 \odot G_2) &= \sum_{i=1}^{n_1}(d_{G_1}(v_i)+n_2)^3 + \sum_{i=1}^{m_1}2^3 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}(d_{G_2}(u_j)+1)^3 \\ &= \sum_{i=1}^{n_1}(d_{G_1}(v_i)^3 + 3n_2d_{G_1}(v_i)^2 + 3{n_2}^2d_{G_1}(v_i) + {n_2}^3) + 8m_1 \\ &\quad + n_1\sum_{j=1}^{n_2}(d_{G_2}(u_j)^3 + 3d_{G_2}(u_j)^2 + 3d_{G_2}(u_j) + 1) \\ &= F(G_1) + 3n_2M_1(G_1) + 6{n_2}^2m_1 + n_1{n_2}^3 + 8m_1 \\ &\quad + n_1(F(G_2) + 3M_1(G_2) + 6m_2 + n_2),\end{aligned}$$

from where the desired result follows. □

Example 1. Using the last theorem, we have
(i) $F(P_n \odot P_m) = nm^3 + 6nm^2 - 6m^2 + 39nm - 22n - 18m - 22$,
(ii) $F(C_n \odot C_m) = nm^3 + 6nm^2 + 39nm + 16n$,
(iii) $F(C_n \odot P_m) = nm^3 + 6nm^2 + 39nm - 22n$.

Subdivision-edge corona

Definition 3. [14] Let G_1 and G_2 be two vertex disjoint graphs. The subdivision-edge corona of G_1 and G_2 is denoted by $G_1 \Theta G_2$ and obtained from $S(G_1)$ and n_1 copies of G_2, all vertex-disjoint, by joining the i-th vertex of $V(G_1)$ to every vertex in the i-th copy of G_2.

From definition, we have the subdivision-edge corona $G_1 \Theta G_2$ has $m_1(1 + n_2) + n_1$ vertices and $m_1(n_2 + m_2 + 2)$ edges. Also, the degree of the vertices of $G_1 \Theta G_2$ are given by
$d_{G_1\Theta G_2}(v_i) = d_{G_1}(v_i)$ for $i = 1, 2, ..., n_1$
$d_{G_1\Theta G_2}(e_i) = 2 + n_2$ for $i = 1, 2, ..., m_1$
$d_{G_1\Theta G_2}(v_j^i) = d_{G_2}(u_j) + 1$ for $i = 1, 2, ..., m_1$ and $j = 1, 2, ..., n_2$.
The subdivision-edge corona of P_4 and P_2 is also depicted in Figure 1. In the following theorem we determine the F-index of subdivision-edge corona of two graphs G_1 and G_2.

Theorem 4. *The F-index of the subdivision-edge corona $G_1 \Theta G_2$ is given by*
$F(G_1 \Theta G_2) = F(G_1) + m_1F(G_2) + 3m_1M_1(G_2) + 6m_1m_2 + m_1(n_2 + 2)^3 + m_1n_2$.

Proof. From the definition of the subdivision-edge corona $G_1 \Theta G_2$, we have

$$\begin{aligned}F(G_1 \Theta G_2) &= \sum_{i=1}^{n_1}d_{G_1}(v_i)^3 + \sum_{i=1}^{m_1}(2+n_2)^3 + \sum_{i=1}^{m_1}\sum_{j=1}^{n_2}(d_{G_2}(u_j)+1)^3 \\ &= F(G_1) + m_1(n_2+2)^3 + m_1\sum_{j=1}^{n_2}(d_{G_2}(u_j)^3 + 3d_{G_2}(u_j)^2 + 3d_{G_2}(u_j) + 1) \\ &= F(G_1) + m_1(n_2+2)^3 + m_1(F(G_2) + 3M_1(G_2) + 6m_2 + n_2),\end{aligned}$$

from where the desired result follows. □

Example 2. The following results are direct consequence of the previous theorem.
(i) $F(P_n \Theta P_m) = nm^3 + 6nm^2 - m^3 - 6m^2 + 39nm - 22n - 39m + 16$,
(ii) $F(C_n \Theta C_m) = nm^3 + 6nm^2 + 39nm + 16n$,
(iii)$F(C_n \Theta P_m) = nm^3 + 6nm^2 + 39nm - 22n$.

Subdivision-vertex neighborhood corona

Definition 5. [15] For two vertex disjoint graphs G_1 and G_2, the subdivision-vertex neighborhood corona of G_1 and G_2 is denoted by $G_1 \boxdot G_2$ and obtained from $S(G_1)$ and n_1 copies of G_2, all vertex-disjoint, by joining the neighbors of the i-th vertex of $V(G_1)$ to every vertex in the i-th copy of G_2.

Let $V(G_1) = \{v_1, v_2, ..., v_{n_1}\}$, $I(G_1) = \{e_1, e_2, ..., e_{m_1}\}$ and $V(G_2) = \{v_1, v_2, ..., v_{n_2}\}$. Also, let the vertex set of the i-th copy of G_2 is denoted by $V(G_2^i) = \{u_1^i, u_2^i, ..., u_{n_2}^i\}$, for $i = 1, 2, ..., n_1$. Then $V(G_1 \boxdot G_2) = V(G_1) \cup I(G_1) \cup [V^1(G_2) \cup V^2(G_2) \cup ... \cup V^{n_1}(G_2)]$. The degree of the vertices of $G_1 \boxdot G_2$ are given by
$d_{G_1 \boxdot G_2}(v_i) = d_{G_1}(v_i)$ for $i = 1, 2, ..., n_1$
$d_{G_1 \boxdot G_2}(e_i) = 2n_2 + 2$ for $i = 1, 2, ..., m_1$
$d_{G_1 \boxdot G_2}(v_j^i) = d_{G_2}(u_j) + d_{G_1}(v_i)$ for $i = 1, 2, ..., n_1$ and $j = 1, 2, ..., n_2$.
The subdivision-vertex neighborhood corona of P_4 and P_2 is given in Figure 2. In the following theorem we determine the F-index of subdivision-vertex neighborhood corona of two graphs.

Theorem 6. *The F-index of* $G_1 \boxdot G_2$ *is given by*
$F(G_1 \boxdot G_2) = (1 + n_2)F(G_1) + n_1 F(G_2) + 6m_1 M_1(G_2) + 6m_2 M_1(G_1) + 8m_1(n_2 + 1)^3$.

Proof. From definition of $G_1 \boxdot G_2$, we have

$$\begin{aligned} F(G_1 \boxdot G_2) &= \sum_{i=1}^{n_1} d_{G_1}(v_i)^3 + \sum_{i=1}^{m_1} (2n_2 + 2)^3 + \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} (d_{G_2}(u_j) + d_{G_1}(v_i))^3 \\ &= F(G_1) + m_1(2n_2 + 2)^3 \\ &\quad + \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} (d_{G_2}(u_j)^3 + 3d_{G_2}(u_j)^2 d_{G_1}(v_i) + 3d_{G_2}(u_j) d_{G_1}(v_i)^2 + d_{G_1}(v_i)^3) \\ &= F(G_1) + 8m_1(n_2 + 1)^3 + n_1 F(G_2) + 6m_1 M_1(G_2) + 6m_2 M_1(G_1) + n_2 F(G_1), \end{aligned}$$

from where the desired result follows. □

Example 3. The following results are follows from the last theorem.
(i) $F(P_n \boxdot P_m) = 8m^3(n - 1) + 24m^2(n - 1) + 88nm - 58n - 98m + 50$,
(ii) $F(C_n \boxdot C_m) = 8nm^3 + 24nm^2 + 88nm + 16n$,
(iii) $F(C_n \boxdot P_m) = 8nm^3 + 24nm^2 + 88nm - 58n$.

Subdivision-edge neighborhood corona

Definition 7. [15] For two vertex disjoint graphs G_1 and G_2, the subdivision-edge neighborhood corona of G_1 and G_2 is denoted by $G_1 \diamond G_2$ and obtained from $S(G_1)$ and n_1 copies of G_2, all vertex-disjoint, by joining the neighbors of the i-th vertex of $V(G_1)$ to every vertex in the i-th copy of G_2.

Let $V(G_1) = \{v_1, v_2, ..., v_{n_1}\}$, $I(G_1) = \{e_1, e_2, ..., e_{m_1}\}$ and $V(G_2) = \{v_1, v_2, ..., v_{n_2}\}$. Also, let the vertex set of the i-th copy of G_2 is denoted by $V(G_2^i) = \{u_1^i, u_2^i, ..., u_{n_2}^i\}$, for $i = 1, 2, ..., m_1$. Then the vertex partition of $G_1 \diamond G_2$ is given by $V(G_1 \diamond G_2) = V(G_1) \cup I(G_1) \cup [V^1(G_2) \cup V^2(G_2) \cup ... \cup V^{m_1}(G_2)]$. The degree of the vertices of $G_1 \diamond G_2$ are given by
$d_{G_1 \diamond G_2}(v_i) = (n_2 + 1)d_{G_1}(v_i)$ for $i = 1, 2, ..., n_1$
$d_{G_1 \diamond G_2}(e_i) = 2$ for $i = 1, 2, ..., m_1$
$d_{G_1 \diamond G_2}(v_j^i) = d_{G_2}(u_j) + 2$ for $i = 1, 2, ..., n_1$ and $j = 1, 2, ..., n_2$.
The subdivision-edge neighborhood corona of P_4 and P_2 is given in Figure 2. In the following theorem we determine the F-index of subdivision-edge neighborhood corona of two graphs G_1 and G_2.

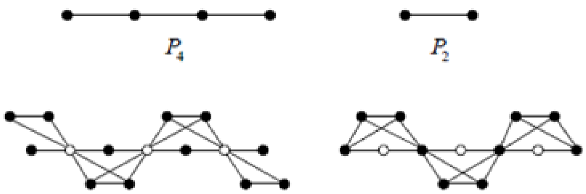

Figure 2: Subdivision-vertex and subdivision-edge neighborhood corona products of P_4 and P_2.

Theorem 8. *The F-index of $G_1 \diamond G_2$ is given by*
$F(G_1 \diamond G_2) = (n_2+1)^3F(G_1) + 8m_1 + n_1F(G_2) + 6n_1M_1(G_2) + 24n_1m_2 + 8n_1n_2.$

Proof. From definition of subdivision-edge neighborhood corona product of two graphs we have

$$\begin{aligned} F(G_1 \diamond G_2) &= \sum_{i=1}^{n_1}(n_2+1)^3 d_{G_1}(v_i)^3 + \sum_{i=1}^{m_1} 2^3 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}(d_{G_2}(u_j)+2)^3 \\ &= (n_2+1)^3F(G_1) + 8m_1 + \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}(d_{G_2}(u_j)^3 + 6d_{G_2}(u_j)^2 + 12d_{G_2}(u_j) + 8) \\ &= (n_2+1)^3F(G_1) + 8m_1 + n_1F(G_2) + 6n_1M_1(G_2) + 24n_1m_2 + 8n_1n_2 \end{aligned}$$

Hence we get the desired result. □

Example 4. From the last theorem we have the following results.
(i) $F(P_n \diamond P_m) = 8nm^3 - 14m^3 + 24nm^2 - 42m^2 + 88nm - 58n - 42m - 22$,
(ii) $F(C_n \diamond C_m) = 8nm^3 + 24nm^2 + 88nm + 16n$,
(iii) $F(C_n \diamond P_m) = 8nm^3 + 24nm^2 + 88nm - 58n$.

The vertex-edge corona

Definition 9. [16] The vertex-edge corona of two graphs G_1 and G_2 is denoted by $G_1 \otimes G_2$, is the graph obtained by taking one copy of G_1, n_1 copies of G_2 and also m_1 copies of G_2, then joining the i-th vertex of G_1 to every in the i-th vertex copy of G_2 and also joining the end vertices of j-th edge of G_1 to every vertex in the j-th edge copy of G_2 , where $1 \le i \le n_1$ and $1 \le j \le m_1$.

Let the vertex set of the j-th edge copy of G_2 is denoted by $V_{j_e}(G_2) = \{u_{j1}, u_{j2}, ..., u_{jn_2}\}$ and the vertex set of the i-th vertex copy of G_2 is denoted by $V_{i_v}(G_2) = \{w_{i1}, w_{i2}, ..., w_{in_2}\}$. Also let us denote the edge set of the j-th edge and i-th vertex copy of G_2 by $E_{j_e}(G_2)$ and $E_{i_v}(G_2)$ respectively. From definition we have the vertex-edge corona $G_1 \otimes G_2$ has $m_1 + m_1(m_2 + 2n_2) + n_1(n_2 + m_2)$ edges and $n_1 + n_2(n_1 + m_1)$ vertices. Also, the degree of the vertices of $G_1 \otimes G_2$ are given by
$d_{G_1 \otimes G_2}(v_i) = (n_2+1)d_{G_1}(v_i) + n_2, \forall v_i \in V(G_1)$
$d_{G_1 \otimes G_2}(u_{ij}) = d_{G_2}(u_j) + 2, \forall u_{ij} \in V_{i_e}(G_2)$
$d_{G_1 \otimes G_2}(w_{ij}) = d_{G_2}(w_j) + 1, \forall w_{ij} \in V_{i_e}(G_2).$
In the following theorem we determine the F-index of vertex-edge corona of two graphs G_1 and G_2.

Theorem 10. *The F-index of $G_1 \otimes G_2$ is given by*
$F(G_1 \otimes G_2) = (n_2+1)^3F(G_1) + (n_1+m_1)F(G_2) + 3n_2(n_2+1)^2M_1(G_1)$
$+3(n_1+2m_1)M_1(G_2) + 6{n_2}^2m_1(n_2+1) + n_1{n_2}^3 + 6n_1m_2 + 8n_2m_1 + 24m_1m_2 + n_1n_2.$

Proof. From definition of vertex-edge corona of graphs, we have

$$\begin{aligned} F(G_1 \otimes G_2) &= \sum_{v_i \in V(G_1)} d_G(v_i)^3 + \sum_{e_i \in E(G_1)} \sum_{u_{ij} \in V(G_2)} d_G(u_{ij})^3 + \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V(G_2)} d_G(w_{ij})^3 \\ &= A_1 + A_2 + A_3, \ (say). \end{aligned}$$

Now to calculate the contribution of A_1, we have

$$\begin{aligned} A_1 &= \sum_{v_i \in V(G_1)} d_G(v_i)^3 \\ &= \sum_{v_i \in V(G_1)} [(n_2+1)d_{G_1}(v_i) + n_2]^3 \\ &= \sum_{v_i \in V(G_1)} [(n_2+1)^3 d_{G_1}(v_i)^3 + 3n_2(n_2+1)^2 d_{G_1}(v_i)^2 + 3{n_2}^2(n_2+1)d_{G_1}(v_i) + {n_2}^3] \\ &= (n_2+1)^3 F(G_1) + 3n_2(n_2+1)^2 M_1(G_1) + 6{n_2}^2 m_1(n_2+1) + n_1 {n_2}^3 \end{aligned}$$

Also, we have the contribution of A_2 as

$$\begin{aligned} A_2 &= \sum_{e_i \in E(G_1)} \sum_{u_{ij} \in V(G_2)} d_G(u_{ij})^3 \\ &= \sum_{e_i \in E(G_1)} \sum_{u_{ij} \in V(G_2)} [d_{G_2}(u_{ij}) + 2]^3 \\ &= \sum_{e_i \in E(G_1)} \sum_{u_j \in V_e(G_2)} [d_{G_2}(u_j)^3 + 6d_{G_2}(u_j)^2 + 12d_{G_2}(u_j) + 8] \\ &= m_1[F(G_2) + 6M_1(G_2) + 24m_2 + 8n_2] \end{aligned}$$

Similarly, we get the contribution of A_3 as follows.

$$\begin{aligned} A_3 &= \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V(G_2)} d_G(w_{ij})^3 \\ &= \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V_{i_v}(G_2)} [d_{G_2}(w_{ij}) + 1]^3 \\ &= \sum_{v_i \in V(G_1)} \sum_{w_{ij} \in V_{i_v}(G_2)} [d_{G_2}(w_{ij})^3 + 3d_{G_2}(w_{ij})^2 + 3d_{G_2}(w_{ij}) + 1] \\ &= n_1[F(G_2) + 3M_1(G_2) + 6m_2 + n_2] \end{aligned}$$

Adding A_1, A_2 and A_3, we get the desired result. □

Example 5. From the last theorem we have the following results.

(i) $F(P_n \otimes P_m) = 2(m+1)^3(4n-7) + 2(2n-1)(4m-7) + 6m(m+1)^2(2m-3)+$ $+6(3n-2)(2m-3) + 6m^2(n-1)(m+1) + nm^3 + 6(m-1)(5n-4) + 9nm - 8m,$

(ii) $F(C_n \otimes C_m) = 27nm^3 + 54nm^2 + 127nm + 8n,$

(iii) $F(C_n \otimes P_m) = 27nm^3 + 54nm^2 + 119nm - 104n.$

Conclusion

In this work, we compute F-index of several types of corona product of two graphs such as subdivision-vertex corona, subdivision-edge corona, subdivision-vertex neighborhood corona, subdivision-edge neighborhood corona and vertex-edge corona. As an application we derive some explicit expressions of corona products of some particular graphs. For further study, other topological indices of these corona product of graphs and for can be computed.

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