

## Operations on Semigraphs

Prabhakar R. Hampiholi<sup>1, a</sup>, Meenal M. Kaliwal<sup>2, b \*</sup>

<sup>1</sup>Professor, Head of the Department, Department of Mathematics, Gogte Institute of Technology, Udyambag, Belagavi, Karnataka, India

<sup>2</sup>Assistant Professor, Department of Mathematics, KLS Vishwanathrao Deshpande Rural Institute of Technology, Affiliated to VTU, Haliyal-581329, Karnataka, India

<sup>a</sup>prhampi@yahoo.co.in, <sup>b</sup>mmkmeenal@gmail.com

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**Abstract.** In this paper the structural equivalence of union, intersection ring sum and decomposition of semigraphs are explored by using the various types of isomorphisms such as isomorphism, ev-isomorphism, a-isomorphism and e-isomorphism for  $G_e$ ,  $G_a$  and  $G_{ca}$ . We establish various types of binary operations in semigraphs.

### 1. Introduction

Many binary operations have been defined in Graph Theory [7] to derive its properties using set theoretical terminology. Boolean Operations on Graphs was introduced by Frank Harary and W.G. Wilcox [8]. Further many operations such as composition [9], Kronecker product of graphs and lexicographic products were enhanced by G. Sabidussi [10, 11] and P.M. Weichsel [17]. These Boolean operations provide an insight for investigating the structural properties of the newly generated graphs from a pair of simple graphs.

The concept of Semigraph was first introduced by E.Sampathkumar [6], which is enriched by variety of properties enjoyed not only by the vertices but also by the edges. Some of the concepts such as domination in semigraphs was introduced by S.S. Kamath, R.S. Bhat and S.R. Hebbar [21, 22], Bipartite theory of semigraphs and Hyper domination in semigraphs by Y.B.Venkatakrishnan and V. Swaminathan [23, 24], (m,e)-domination in semigraphs by S. Gomathi [20], Adjacency matrix and Incidence matrix of a semigraph by C.M.Deshpande, Y.S. Gaidhani and B.P.Athawale [3, 4], Factorization and Genus of semigraphs by P. Das and Surajith Kr. Nath [18, 19], Partial edge incidence matrix and Strong Circuit matrix and Strong Path matrix by P.R. Hampiholi and J.P. Kitturkar [15, 16]. The application of semigraph in DNA splicing was introduced by K. Thiagarajan, J. Padmashree and S. Jeyabharathi [12].

Semigraphs provide a wide scope for defining various types of fundamental operations and explore its hidden properties. Semigraphs mainly contains two types of edges called subedges and partial edges which play a very prominent role in the application of resolving traffic problems, representing family relationships. We employ some of these fundamental operations in Semigraphs with respect to partial edge, subedges and equal edges. The properties of partial edges, subedges and equal edges give rise to various types of union, intersection, decomposition and ring sum concepts in semigraphs. The main objective of this paper is to define full-edge operation, end-vertex operation and subedge operation. We can find applications of these operations in Eulerian semigraphs [5].

### 2. Preliminaries

**Definition 1:** [6] A semigraph  $G$  is a pair  $(V, X)$  where  $V$  is a nonempty set whose elements are called vertices of  $G$ , and  $X$  is a set of n-tuples, called edges of  $G$ , of distinct vertices  $n \geq 2$ , satisfying the following conditions:

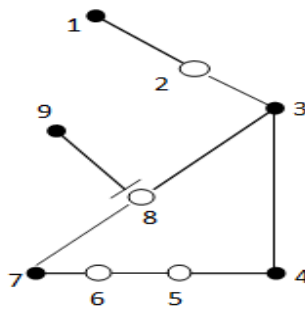
SG1: Any two edges have atmost one vertex in common.

SG2: Two edges  $(u_1, u_2, \dots, u_n)$  and  $(v_1, v_2, \dots, v_m)$  are considered to be equal if and only if

- (1)  $m = n$  and
- (2) Either  $u_i = v_i$  for  $1 \leq i \leq n$ , or  $u_i = v_{n-i+1}$  for  $1 \leq i \leq n$ .

Thus, the edge  $E = (u_1, u_2, \dots, u_n)$  is same as the edge  $(u_n, u_{n-1}, \dots, u_1)$ .  $u_1$  and  $u_n$  are called the end vertices of  $E$ , while  $u_2, u_3, \dots, u_{n-1}$  are called the middle vertices of  $E$ . Also, a vertex which is a middle vertex for one edge and an end vertex of another edge is said to be an  $(m, e)$ -vertex. The middle vertex is drawn as a hollow circle and an  $(m, e)$ -vertex is drawn as a hollow circle with a small tangent drawn to it indicating it is an end vertex of the other edge.

**Example 1:** Let  $G=(V, X)$  be a semigraph shown below in Figure 1 where  $V = \{1,2,3,4,5,6,7,8\}$  and  $X = \{(1,2,3), (3,4), (4,5,6,7), (7,8,3), (8,9)\}$  shown below in Figure 1. The vertices 1 and 3 are the end vertices of the edge  $(1,2,3)$  and 2 is the middle vertex of this edge.



**Figure 1.**

Also vertex 8 is the  $(m, e)$  vertex of the edge  $(7,8,3)$ . Vertex 4 is adjacent to the vertex 6 whereas vertex 4 is consecutively adjacent to vertex 5.

**Definition 3:** [6] A *partial edge* or *fp edge*  $E$  is a  $j-i+1$ -tuple  $E(v_i, v_j) = (v_i, v_{i+1}, \dots, v_j)$ , where  $1 \leq i \leq n$ .

**Definition 4:** [6] A *subedge* or *fs-edge*  $E = (v_1, v_2, \dots, v_n)$  is a  $k$ -tuple  $E' = (v_{i_1}, v_{i_2}, \dots, v_{i_k})$  where  $1 \leq i_1 < i_2 < \dots < i_k \leq n$  or  $1 \leq i_k < i_{k-1} < \dots < i_1 \leq n$ .

**Definition 5:** [6] A *full edge* or *f edge* is any edge of a semigraph.

**Definition 6:** [6] Two edges are said to be *equal* if they have the same cardinality, that is if both the edges contain the same number of vertices.

**Definition 7:** [6] A semigraph  $G' = (V', X')$  is a subsemigraph of a semigraph  $G = (V, X)$  if  $V' \subseteq V$  and the edges in  $G'$  are the subedges of the edges in  $G$ .

**Definition 8:** [1] A *clique* of a semigraph  $G$  is a complete subsemigraph of  $G$ .

**Definition 9:** [6] The *adjacency graph* denoted by  $G_a$  is a graph in which two vertices in  $G_a$  are adjacent if they are adjacent in  $G$ .

**Definition 10:** [6] The *consecutive adjacency graph*  $G_{ca}$  is a graph in which two vertices in  $G_{ca}$  are adjacent if they are consecutively adjacent vertices in  $G$ .

**Definition 11:** [6] The *end vertex graph* denoted by  $G_e$  is a graph in which two vertices in  $G_e$  are adjacent if they are end vertices of an edge in  $G$ .

**Definition 12:** [6] The union  $G_1 \cup G_2$  of two adjacency disjoint semigraphs  $G_1 = (V_1, X_1)$  and  $G_2 = (V_2, X_2)$  is a semigraph with vertex set  $V_1 \cup V_2$  and edge set  $X_1 \cup X_2$ .

**Definition 13:** [6] A semigraph  $G^c = (V, X)$  with vertex set  $V$  is called a complement of a semigraph  $G = (V, X)$  if for any two vertices  $u$  and  $v$  in  $G$ ,  $u$  and  $v$  are adjacent in  $G^c$  if they are non adjacent in  $G$ .

**Definition 14:** [6] Let  $G_1 = (V_1, X_1)$  and  $G_2 = (V_2, X_2)$  be two semigraphs and  $f$  be a bijection from  $V_1$  to  $V_2$ . Let  $E = (v_1, v_2, \dots, v_n)$  be an edge in  $G_1$ . Then

1.  $f$  is an *isomorphism* if  $(f(v_1), f(v_2), \dots, f(v_n))$  is an edge in  $G_2$ .
2.  $f$  is an *end vertex isomorphism (ev-isomorphism)* if  $(f(v_1), f(v_2), \dots, f(v_n))$  is an edge in  $G_2$  with end vertices  $f(v_1)$  and  $f(v_n)$ .
3.  $f$  is an *edge isomorphism (e-isomorphism)* if the set  $(f(v_1), f(v_2), \dots, f(v_n))$  forms an edge in  $G_2$ .
4.  $f$  is an *adjacency isomorphism (a-isomorphism)* if adjacent vertices in  $G_1$  are mapped onto adjacent vertices in  $G_2$ .

**Theorem 15:** [6] A semigraph  $G$  is  $p$ -eulerian if and only if, every vertex in  $G$  has an even degree.

### 3. Main Results

Now we define union, intersection and ring sum operations on two semigraphs having disjoint edges or equal edges.

#### 3.1. Union of Two Semigraphs

**Definition 3.1.1:** If two semigraphs  $G_1$  and  $G_2$  are such that any edge in  $G_1$  is either distinct with any edge in  $G_2$  or the edges are equal then  $G_1 \cup G_2$  is a semigraph with the vertex set  $V_1 \cup V_2$  and edge set  $X_1 \cup X_2$ .

**Example 3.1.2:**

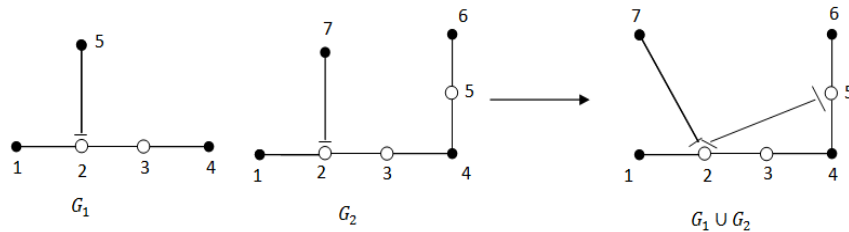


Figure 2.

The following observations are obvious.

**Remarks 3.1.3:**

- i) For any semigraph  $G$ ,  $G \cup G = G$ .
- ii) For any semigraph  $G$ ,  $G \cup G^c \cong K(G)$  where  $K(G)$  is a complete semigraph containing both  $G$  and  $G^c$ .
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \cup G_2 = G_2 \cup G_1$ .
- iv) For semigraphs  $G_1$ ,  $G_2$  and  $G_3$ ,  $G_1 \cup (G_2 \cup G_3) = (G_1 \cup G_2) \cup G_3$ .

With this we proceed to prove the main results.

**Theorem 3.1.4:** If  $G_a, G_e$  and  $G_{ca}$  represent the adjacency graph, end vertex graph and consecutive adjacency graph respectively, then for any two semigraphs  $G'$  and  $G''$ , we have

- i)  $G'_a \cup G''_a \cong (G' \cup G'' )_a$
- ii)  $G'_e \cup G''_e \cong (G' \cup G'' )_e$
- iii)  $G'_{ca} \cup G''_{ca} \cong (G' \cup G'' )_{ca}$

**Proof:** i) By the definition 3.1, the vertex set of  $G'_a \cup G''_a$  is same as the vertex set of  $(G' \cup G'' )_a$ . Let,  $E(u_1, u_n) = (u_1, u_2, \dots, u_n) \in E(G' \cup G'')$ . This implies either  $E \in E(G')$  or  $E \in E(G'')$  or both. Without loss of generality, let  $E \in E(G')$ . Then, the edge  $E$  corresponds to the clique say  $C$  in

$G'_\alpha$ . By definition 2.9, this implies that for each edge  $E \in E(G' \cup G'')$  there corresponds a clique  $C$  in  $G'_\alpha$  or  $G''_\alpha$  or both. Hence, clique  $C \in G'_\alpha \cup G''_\alpha$ . Again by the definition 2.9, for the same edge  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n) \in E(G' \cup G'')$ , there exists a clique  $C \in (G' \cup G'')_\alpha$ . Hence, for each edge  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n)$  of  $G' \cup G''$  there corresponds a clique  $C \in G'_\alpha \cup G''_\alpha$  and  $C \in (G' \cup G'')_\alpha$  on vertices  $u_1, u_2, \dots, \dots, u_n$ .

Now to show the existence of isomorphism between  $G'_\alpha \cup G''_\alpha$  and  $(G' \cup G'')_\alpha$  we establish the following result. Let  $E = (E_1, E_2, \dots, \dots, E_k)$  be the edges of  $G' \cup G''$  and  $C = (C_1, C_2, \dots, \dots, C_k)$  be the cliques in  $G'_\alpha \cup G''_\alpha$ .

Let,  $f: E \rightarrow C$  be a function defined as  $f(E_i) = C_i$ . Then, we have the following

- (i)  $|E_i| = |C_i|$  (established above)
- (ii) If  $E_i$  belongs to both  $E(G'_\alpha)$  and  $E(G''_\alpha)$  then, there exists one common clique  $C_i \in C$
- (iii) If,  $E_i \neq E_j$  with  $|E_i \cap E_j| = 1$ , then  $f(E_i) = C_i$  and  $f(E_j) = C_j$  with  $|C_i \cap C_j| = 1$ . Also if  $E_i \neq E_j$  then we claim that  $C_i \neq C_j$  or  $f(E_i) \neq f(E_j)$  for otherwise if  $C_i = C_j$  then  $|C_i| = |E| - 1$ , which is a contradiction to (i). Therefore, by definition 2.14  $f$  is an *adjacency-isomorphism*. Hence,  $G'_\alpha \cup G''_\alpha \cong (G' \cup G'')_\alpha$ .

ii) By the definition 3.1, the vertex set of  $G'_\theta \cup G''_\theta$  is same as the vertex set of  $(G' \cup G'')_\theta$ .

Let,  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n) \in E(G' \cup G'')$ . This implies either  $E \in E(G')$  or  $E \in E(G'')$  or both. Without loss of generality, let  $E \in E(G')$ . Let  $u_1$  and  $u_n$  be the end vertices of the edge  $E$  in  $G'$ . Hence by the definition 2.11,  $u_1$  and  $u_n$  are the end vertices of the edge  $E'$  in  $G'_\theta$ . This implies that for each edge  $E \in E(G' \cup G'')$ , there corresponds a complete graph of order 2 that is  $K_2$  in  $G'_\theta$  or  $G''_\theta$  or both. Hence, for each edge  $E$  there corresponds  $K_2 \in G'_\theta \cup G''_\theta$ . Again by the definition 2.11, for the same edge  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n) \in E(G' \cup G'')$ , there corresponds complete graph  $K_2 \in (G' \cup G'')_\theta$ . Hence, for each edge  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n)$  of  $(G' \cup G'')$ , there corresponds a complete graph  $K_2 \in G'_\theta \cup G''_\theta$  and  $K_2 \in (G' \cup G'')_\theta$  with the end vertices  $u_1$  and  $u_n$ .

Let,  $E = (E_1, E_2, \dots, \dots, E_k)$  be the set of edges and  $M = \{m_1, m_2, \dots, \dots, m_l\}$  be the set of middle vertices  $G' \cup G''$ . Also, let  $E_\theta = (E_{\theta_1}, E_{\theta_2}, \dots, \dots, E_{\theta_k})$  be the set of edges and  $M_\theta = (m_{\theta_1}, m_{\theta_2}, \dots, \dots, m_{\theta_l})$  be the set of isolated vertices in  $G'_\theta \cup G''_\theta$ . Clearly,  $|M| = |M_\theta|$  and  $|E| = |E_\theta|$ .

Now an isomorphism between  $G'_\theta \cup G''_\theta$  and  $(G' \cup G'')_\theta$  will be established.

Let  $f: (E \cup M) \rightarrow (E_\theta \cup M_\theta)$  be the function defined as

$$f(E) = E_\theta, \text{ if } |E| = 2$$

$$= E_\theta \cup \left( \bigcup_{m_i \in M} m_i \right), \quad 1 \leq i \leq l \text{ if } |E| > 2$$

We claim that, if  $E_i \neq E_j$  then  $f(E_i) \neq f(E_j)$  for otherwise, if  $E_i \neq E_j$  and  $f(E_i) = f(E_j)$  then  $|E_\theta| = |E| - 1$ . Therefore, by definition 2.14  $f$  is an *end vertex isomorphism*.

Hence,  $G'_\theta \cup G''_\theta \cong (G' \cup G'')_\theta$ .

iii) By the definition 3.1, the vertex set of  $G'_{ca} \cup G''_{ca}$  is same as the vertex set of  $(G' \cup G'')_{ca}$ .

Let,  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n) \in E(G' \cup G'')$ . This implies either  $E \in E(G')$  or  $E \in E(G'')$  or both. Without loss of generality, let  $E \in E(G')$ . Then, the edge  $E$  corresponds to a path  $P_n$  in  $G'_{ca}$ . This implies that for each edge  $E \in E(G' \cup G'')$  there is a path  $P_n$  in  $G'_{ca}$  or  $G''_{ca}$  or both. Hence, by the definition 2.10, path  $P_n \in G'_{ca} \cup G''_{ca}$ . Similarly, for the edge  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n) \in E(G' \cup G'')$ , there exists a path  $P_n \in (G' \cup G'')_{ca}$ . Hence, for each edge  $E(u_1, u_n) = (u_1, u_2, \dots, \dots, u_n)$  of  $(G' \cup G'')$ , there corresponds a path  $P_n \in G'_{ca} \cup G''_{ca}$  and by the definition 2.10, the same path  $P_n \in (G' \cup G'')_{ca}$  with vertices  $u_1, u_2, \dots, \dots, u_n$  in order. By the definition 2.10 and 2.14, the consecutive adjacency graph  $G_{ca}$  of a semigraph  $G$  resembles a graph. Hence the establishment of isomorphism between  $G'_{ca} \cup G''_{ca}$  and  $(G' \cup G'')_{ca}$  is trivial.

Hence,  $G'_{ca} \cup G''_{ca} \cong (G' \cup G'')_{ca}$ .

**Definition 3.1.5: Intersection of Two Semigraphs**

If two semigraphs  $G_1$  and  $G_2$  are such that any edge in  $G_1$  has either at most one vertex in common with any edge in  $G_2$  or the edges are equal then  $G_1 \cap G_2$  is a semigraph with vertex set  $V_1 \cap V_2$  and edge set  $X_1 \cap X_2$ .

**Example 3.1.6:**

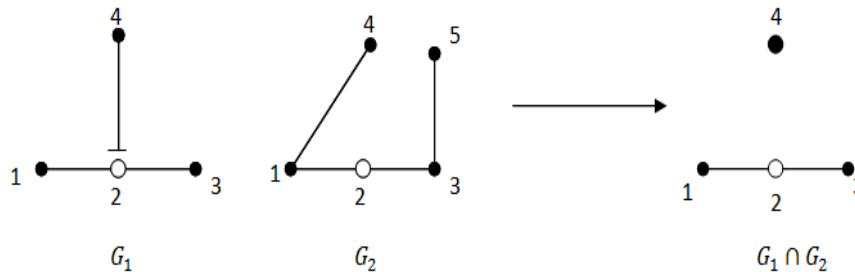


Figure 3.

**Remarks 3.1.7:**

- i) For any semigraph  $G$ ,  $G \cap G = G$ .
- ii) For any semigraph  $G$ ,  $G \cap G^c$  is a null graph.
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \cap G_2 = G_2 \cap G_1$ .
- iv) For any semigraphs  $G_1, G_2$  and  $G_3$ ,  $G_1 \cap (G_2 \cap G_3) = (G_1 \cap G_2) \cap G_3$ .
- v) For any semigraphs  $G_1, G_2$  and  $G_3$ ,
  - a)  $G_1 \cap (G_2 \cup G_3) = (G_1 \cap G_2) \cup (G_1 \cap G_3)$
  - b)  $G_1 \cup (G_2 \cap G_3) = (G_1 \cup G_2) \cap (G_1 \cup G_3)$ .

**Theorem 3.1.8:** If  $G_a, G_s$  and  $G_{ca}$  are the adjacency graph, end vertex graph and consecutive adjacency graph respectively of a semigraph  $G$ , then for any two semigraphs  $G'$  and  $G''$  we have

- i)  $G'_a \cap G''_a \cong (G' \cap G'')_a$
- ii)  $G'_s \cap G''_s \cong (G' \cap G'')_s$
- iii)  $G'_{ca} \cap G''_{ca} \cong (G' \cap G'')_{ca}$

**Proof: i)** By the definition 3.5, the vertex set of  $G'_a \cap G''_a$  is same as the vertex set of  $(G' \cap G'')_a$ .  
 Let,  $E = \{u_1, u_2, \dots, u_n\} \in E(G' \cap G'')$ . This implies  $E \in E(G')$  and  $E \in E(G'')$ . Then, by the definition 2.9 the edge  $E$  corresponds to a clique  $C$  in  $(G' \cap G'')_a$ . This implies that for each edge  $E \in E(G' \cap G'')$  there corresponds a clique  $C$  in  $(G' \cap G'')_a$ . Hence clique  $C \in (G' \cap G'')_a$ . Again by the definition 2.9, for the same edge  $E = \{u_1, u_2, \dots, u_n\} \in E(G' \cap G'')$ , there corresponds a clique  $C \in G'_a \cap G''_a$ . Hence for each edge  $E = \{u_1, u_2, \dots, u_n\}$  of  $G' \cap G''$  there corresponds a clique  $C \in G'_a \cap G''_a$  and the same clique  $C \in (G' \cap G'')_a$  of vertices  $u_1, u_2, \dots, u_n$ .

It remains to establish the isomorphism between  $G'_a \cap G''_a$  and  $(G' \cap G'')_a$ .

Let  $E = \{E_i \text{ and } E_j / E_i \in G', E_j \in G'', |E_i \cap E_j| > 1\}$  be the edge set and  $U = \{u_i / u_i \in E_i \text{ and } E_j, E_i \in G', E_j \in G'' \text{ and } E_i \neq E_j\}$  be the set of vertices. Let  $C = \{C_i / C_i \in (G' \cap G'')_a\}$  be the set of cliques and  $U' = \{u'_i / u'_i \in (G' \cap G'')_a \text{ with } u_i \in U\}$  be the set of isolated vertices. Let,  $f: (E \cup U) \rightarrow (C \cup U')$  be a function defined as  $f(E_i) = C_i$  and  $f(u_i) = u'_i$ . Clearly,  $|E_i| = |C_i|$  and  $|u_i| = |u'_i|$ .

Now,  $|E \cup U| = |C \cup U'|$ . We claim that,  $f(E_i) \neq f(E_j)$  if  $E_i \neq E_j$ . For otherwise, if  $f(E_i) = f(E_j)$  and  $E_i \neq E_j$  then  $|C_i| = |E| - 1$  which is a contradiction. Also, if  $u_r$  is the common vertex to  $E_r, E_s \in G'$  and  $E_r', E_s' \in G''$  then  $u_r'$  will be the common vertex to the cliques  $C_r$  and  $C_s \in (G' \cap G'')_a$ . Hence adjacency of two edges in  $G'$  and  $G''$  is preserved as adjacency of two cliques in  $(G' \cap G'')_a$ . Hence,  $G'_a \cap G''_a \cong (G' \cap G'')_a$ .

Proofs of (ii) and (iii) are similar to (i).

### Definition 3.1.9: Ring Sum of Two Semigraphs

If two semigraphs  $G_1$  and  $G_2$  are such that any edge in  $G_1$  has either at most one vertex in common with any edge in  $G_2$  or any one of the edge in  $G_1$  is equal to any one of the edge in  $G_2$  (equal edges) then the ring sum of semigraphs  $G_1$  and  $G_2$  denoted by  $G_1 \oplus G_2$  is a semigraph with the vertex set  $V_1 \cup V_2$  and the edges are the full edges present either in  $G_1$  or  $G_2$  but not in both. That is  $G_1 \oplus G_2 = \{(V_1 \cup V_2), [(E_1 \cup E_2) - (E_1 \cap E_2)]\}$

### Example 3.1.10:

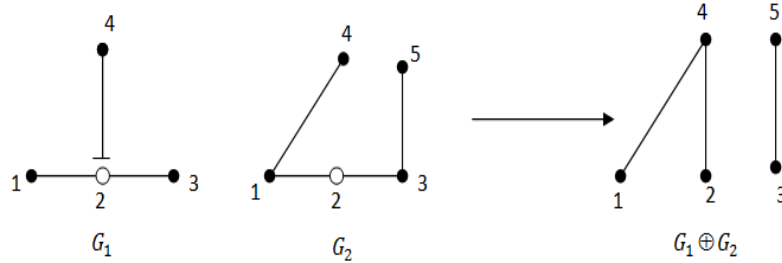


Figure 4.

### Remarks 3.1.11:

- i) For any semigraph  $G$  on  $n$  vertices,  $G \oplus G = \overline{K_n}$ , null graph.
- ii) For any semigraph  $G$  on  $n$  vertices,  $G \oplus \overline{K_n} = G$ .
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \oplus G_2 = G_2 \oplus G_1$ .
- iv) For any semigraphs  $G_1, G_2$  and  $G_3$ ,  $G_1 \oplus (G_2 \oplus G_3) = (G_1 \oplus G_2) \oplus G_3$ .

**Theorem 3.1.12:** If  $G_a, G_e$  and  $G_{ca}$  are the adjacency graph, end vertex graph and consecutive adjacency graph respectively of a semigraph  $G$ , then for any two semigraphs  $G'$  and  $G''$  we have

- i)  $G'_a \oplus G''_a \cong (G' \oplus G'' )_a$
- ii)  $G'_e \oplus G''_e \cong (G' \oplus G'' )_e$
- iii)  $G'_{ca} \oplus G''_{ca} \cong (G' \oplus G'' )_{ca}$

**Proof:** i) By the definition 3.9, the vertex set of  $G'_a \oplus G''_a$  is same as the vertex set of  $(G' \oplus G'' )_a$ .

Let  $E_i = \{E_i/E_i \in G', i = 1, 2, \dots, k\}$  and  $E_j = \{E_j/E_j \in G'', j = 1, 2, \dots, m\}$  be the set of edges. By definition 2.9, for each edge  $E_i$  in  $G'$  there is a clique say,  $C_i$  in  $G'_a$  and for each edge  $E_j$  in  $G''$  there is a clique say,  $C_j$  in  $G''_a$  provided  $E_i \neq E_j$ . This implies  $(C_i \cup C_j) \in (G'_a \oplus G''_a)$ . Also, if any edge  $E_i$  in  $G'$  is equal to some edge  $E_j$  in  $G''$ , then  $C_i = C_j$  and hence  $(C_i \cap C_j) \notin (G'_a \oplus G''_a)$ . Again by the definition 2.9, for the same set of edges  $E_i$  and  $E_j$ , the corresponding cliques  $C_i$  and  $C_j$  exists in  $G' \oplus G''$  and also in  $(G' \oplus G'' )_a$  for  $E_i \neq E_j$ . Thus, for each set of edges  $E_i$  and  $E_j$  there corresponds cliques  $(C_i \cup C_j) \in (G'_a \oplus G''_a)$  and  $(C_i \cup C_j) \in (G' \oplus G'' )_a$ .

To establish the isomorphism we proceed in the following way.

Let,  $f: \{(E_i \cup E_j) - (E_i \cap E_j)\} \rightarrow \{(C_i \cup C_j) - (C_i \cap C_j)\}$  be a function defined as  $f(E_k) = C_k$ .

The remaining proof is similar as in Theorem 3.1.4.

Proofs of (ii) and (iii) are similar to (i).

### Definition 3.1.13: Decomposition of Two Semigraphs

A semigraph  $G$  is said to be *decomposed* into two subsemigraphs  $G_1$  and  $G_2$  if  $G_1 \cup G_2 = G$  and  $G_1 \cap G_2 = \emptyset$ .

**Definition 3.1.14:** A circuit of a semigraph  $G$  which contains a strong cycle is called a *strong circuit* and is denoted by  $C_s$ .

Similarly we define a *weak circuit* of a semigraph.

**Theorem 3.1.15:** A semigraph  $G$  is  $p$ -eulerian, if and only if it can be decomposed into strong circuits.

**Proof:** Let  $G$  be a  $p$ -eulerian semigraph. Consider the consecutive adjacency graph  $G_{ca}$  of  $G$ . Then the graph  $G_{ca}$  is also an Eulerian graph containing a closed walk. By theorem 2.15 it follows that every vertex of  $G$  and its  $G_{ca}$  is of even degree. Hence, the walk traversed from the vertex  $v$  has to end at the same vertex  $v$ , thus forming a strong circuit  $C_s$  in  $G$ . On removing this strong circuit  $C_s$  from  $G$ , the remaining vertices must also be of even degree. Again the remaining vertices are traversed in a similar manner until all the strong circuits are exhausted. This process is repeated until all the partial edges are traversed which forms a strong circuit. Hence, a semigraph  $G$  can be decomposed into strong circuits.

Conversely, suppose a semigraph  $G$  can be decomposed into strong circuits. That is,  $G_{ca}$  is a union of edge disjoint circuits. It follows that the degree of every vertex in the circuit is even. Hence,  $G_{ca}$  is an Eulerian graph. Thus, semigraph  $G$  is  $p$ -eulerian.

**3.2. End-vertex Union of Semigraphs (ev-union)**

**Definition 3.2.1:** If  $G_1 = (V_1, X_1)$  and  $G_2 = (V_2, X_2)$  be two semigraphs then the end-vertex union of  $G_1$  and  $G_2$  denoted by  $G_1 \cup_{ev} G_2$  is a semigraph with the vertex set  $V_1 \cup V_2$  and the edge set satisfies the following the conditions:

- 1) If  $|E_{X_1} \cap E_{X_2}| \leq 1$ , then  $E_{X_1}$  and  $E_{X_2} \in G_1 \cup_{ev} G_2$  where  $E_{X_1} \in G_1$  and  $E_{X_2} \in G_2$ .
- 2) If  $E_{X_1}$  and  $E_{X_2}$  are edges having the same end vertices and with the same order of common middle vertices with  $|E_{X_1} \cap E_{X_2}| \geq 2$  then  $E = E_{X_1} \cup E_{X_2} \in G_1 \cup_{ev} G_2$  is an edge whose end vertices are the end vertices of  $E_{X_1}$  and  $E_{X_2}$  and the permutations of middle vertices of  $E$  retain the order of vertices in  $E_{X_1}$  and  $E_{X_2}$ .

Clearly, the end-vertex union of semigraphs is not unique.

**Example 3.2.2:**

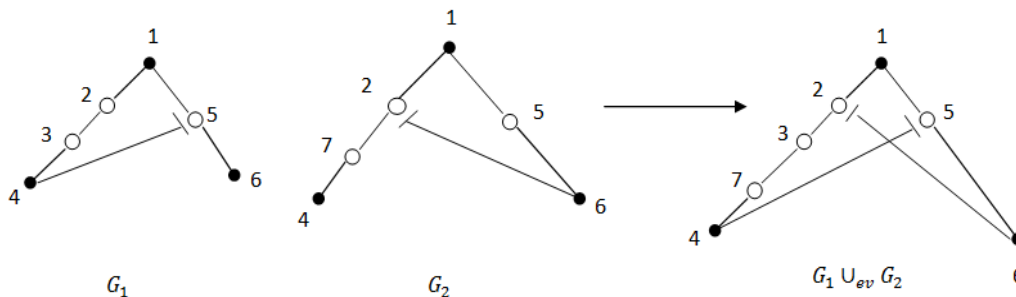


Figure 5.

**Remarks 3.2.3:**

- i) For any semigraph  $G$ ,  $G \cup_{ev} G \cong_{ev} G$ .
- ii) For any semigraph  $G$ ,  $G \cup_{ev} G^c \cong_{ev} K(G)$  where  $K(G)$  is complete semigraph containing both  $G$  and  $G^c$ .
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \cup_{ev} G_2 \cong_{ev} G_2 \cup_{ev} G_1$ .
- iv) For any semigraphs  $G_1, G_2$  and  $G_3$ ,  $G_1 \cup_{ev} (G_2 \cup_{ev} G_3) \cong_{ev} (G_1 \cup_{ev} G_2) \cup_{ev} G_3$

**Theorem 3.2.4:** If  $G_a$  and  $G_e$  are the adjacency graph and the end vertex graphs respectively of a semigraph  $G$ , then for any two semigraphs  $G'$  and  $G''$  we have

- i)  $G'_a \cup_{ev} G''_a \cong (G' \cup_{ev} G'')_a$
- ii)  $G'_e \cup_{ev} G''_e \cong (G' \cup_{ev} G'')_e$

**Proof:** i) By the definition 4.1, the vertex set of  $G'_a \cup_{ev} G''_a$  is same as the vertex set of  $(G' \cup_{ev} G'')_a$ . Let us consider two edges  $E_{X_1} = \{u_1, x, y, u_n\} \in G'$  and  $E_{X_2} = \{u_1, x, y, w, u_n\} \in G''$

having the same end vertices  $u_1$  and  $u_n$ , with atleast one different middle vertex. Then, the end vertex union of  $G'_a \cup_{ev} G''_a$  contains the same clique as that of  $G' \cup_{ev} G''$ , retaining the order of vertices. Hence for each  $E_{X_1} \cup E_{X_2}$ , there corresponds a unique clique denoted by  $C_{E_{X_1} \cup E_{X_2}}$  in  $G'_a \cup_{ev} G''_a$ . Similarly, for the same edges  $E_{X_1}$  and  $E_{X_2}$  with the same end vertices there corresponds a unique clique  $C_{E_{X_1} \cup E_{X_2}}$  in  $(G' \cup_{ev} G'')_a$ , with the same vertex order. Hence,  $C_{E_{X_1} \cup E_{X_2}} \in G'_a \cup_{ev} G''_a$  and also  $C_{E_{X_1} \cup E_{X_2}} \in (G' \cup_{ev} G'')_a$  with the same end vertices.

$$\text{Hence, } G'_a \cup_{ev} G''_a \cong (G' \cup_{ev} G'')_a$$

ii) By the definition 4.1, the vertex set of  $G'_s \cup_{ev} G''_s$  is same as the vertex set of  $(G' \cup_{ev} G'')_s$ .

Let us consider two edges  $E_{X_1} = \{u_1, x, y, u_n\} \in G'$  and  $E_{X_2} = \{y, r, s\} \in G''$ , where  $y$  is an  $(m, e)$  vertex in  $E_{X_1}$ . For every pair of edges  $E_{X_1}$  and  $E_{X_2} \in G'$  or  $G''$ , there exists adjacent complete graphs say  $K_2$  s' in  $G'_s$  or  $G''_s$ . Hence for each pair of edges  $E_{X_1}, E_{X_2} \in (G' \cup_{ev} G'')$ , there corresponds adjacent complete graphs  $K_2$  s' in  $(G' \cup_{ev} G'')_s$ .

Now for the same pair of edges  $E_{X_1}$  and  $E_{X_2} \in (G' \cup_{ev} G'')$ , there corresponds adjacent complete graphs  $K_2$  s' which belong to  $(G' \cup_{ev} G'')_s$ . Hence for each pair of edges  $E_{X_1}$  and  $E_{X_2}$  of  $G'_s \cup_{ev} G''_s$ , there corresponds adjacent complete graphs  $K_2$  s' which belongs to  $G' \cup_{ev} G''$  and also  $K_2$  s' belongs to  $(G' \cup_{ev} G'')_s$  with the same end vertices  $(u_1, u_n)$  and  $(y, s)$ .

### Definition 3.2.5: End-vertex Intersection of Semigraphs (ev-intersection)

If  $G_1 = (V_1, X_1)$  and  $G_2 = (V_2, X_2)$  be two semigraphs then the end-vertex intersection of semigraphs  $G_1$  and  $G_2$  denoted by  $G_1 \cap_{ev} G_2$  is a semigraph with the vertex set  $V_1 \cap V_2$  and the edge set satisfies the following the conditions:

- 1) If  $|E_{X_1} \cap E_{X_2}| \leq 1$ , then  $E_{X_1}$  and  $E_{X_2} \in G_1 \cap_{ev} G_2$  where  $E_{X_1} \in G_1$  and  $E_{X_2} \in G_2$ .
- 2) If  $E_{X_1}$  and  $E_{X_2}$  are edges having the same end vertices and with the same order of common middle vertices with  $|E_{X_1} \cap E_{X_2}| \geq 2$  then  $E = E_{X_1} \cap E_{X_2} \in G_1 \cap_{ev} G_2$  is an edge whose end vertices are the end vertices of  $E_{X_1}$  and  $E_{X_2}$  and the permutations of middle vertices of  $E$  retain the order of vertices in  $E_{X_1}$  and  $E_{X_2}$ .

Clearly, the end-vertex intersection of semigraphs is not unique.

### Example 3.2.6:

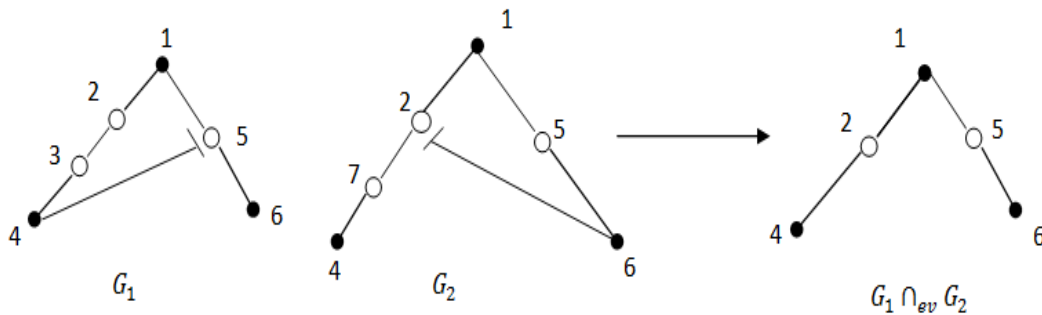


Figure 6.

### Remarks 3.2.7:

- i) For any semigraph  $G$ ,  $G \cap_{ev} G \cong_{ev} G$ .
- ii) For any semigraph  $G$ ,  $G \cap_{ev} G^c$  is a null graph.
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \cap_{ev} G_2 \cong_{ev} G_2 \cap_{ev} G_1$ .
- iv) For any semigraphs  $G_1, G_2$  and  $G_3$ 

$$G_1 \cap_{ev} (G_2 \cap_{ev} G_3) \cong_{ev} (G_1 \cap_{ev} G_2) \cap_{ev} G_3$$
- v) For any semigraphs  $G_1, G_2$  and  $G_3$ 
  - a)  $G_1 \cap_{ev} (G_2 \cap_{ev} G_3) \cong_{ev} (G_1 \cap_{ev} G_2) \cup_{ev} (G_1 \cap_{ev} G_3)$
  - b)  $G_1 \cup_{ev} (G_2 \cap_{ev} G_3) \cong_{ev} (G_1 \cup_{ev} G_2) \cap_{ev} (G_1 \cup_{ev} G_3)$



**Theorem 3.2.8:** If  $G_a$  and  $G_e$  are the adjacency graph and the end vertex graph respectively of a semigraph  $G$ , then for any two semigraphs  $G'$  and  $G''$  we have

- i)  $G'_a \cap_{ev} G''_a \cong (G' \cap_{ev} G'')_a$
- ii)  $G'_e \cap_{ev} G''_e \cong (G' \cap_{ev} G'')_e$

**Proof:** Proof is similar to Theorem 3.1.8.

**Definition 3.2.9: End-vertex Decomposition**

A semigraph  $G$  on vertices  $n$  is said to have an *end vertex decomposition* if it can be decomposed into two subsemigraphs  $G_1$  and  $G_2$  such that  $G_1 \cup_{ev} G_2 = G$  and  $G_1 \cap_{ev} G_2 = \overline{K_n}$ .

**Definition 3.2.10: End-vertex Ring Sum**

The *end-vertex ring sum* of two semigraphs  $G_1$  and  $G_2$  denoted by  $G_1 \oplus_{ev} G_2$  such that the edges  $E_{X_1}$  and  $E_{X_2}$  having the same end vertices is defined as a semigraph with vertex set  $V_1 \cup V_2$  and the edges are the edges present either in  $G_1$  or  $G_2$  but not both.

**Remarks 3.2.11:**

- i) For any semigraph  $G$  on  $n$  vertices,  $G \oplus_{ev} G = \overline{K_n}$ , a null graph.
- ii) For any semigraph  $G$  on  $n$  vertices,  $G \oplus_{ev} \overline{K_n} = G$ .
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \oplus_{ev} G_2 = G_2 \oplus_{ev} G_1$ .
- iv) For any semigraphs  $G_1, G_2$  and  $G_3$ ,  $G_1 \oplus_{ev} (G_2 \oplus_{ev} G_3) = (G_1 \oplus_{ev} G_2) \oplus_{ev} G_3$ .

**Theorem 3.2.12:** If  $G_a$  and  $G_e$  represent the adjacency graph and the end vertex graph respectively of a semigraph  $G$ , then for any two semigraphs  $G'$  and  $G''$  we have

- i)  $G'_a \oplus_{ev} G''_a \cong (G' \oplus_{ev} G'')_a$
- ii)  $G'_e \oplus_{ev} G''_e \cong (G' \oplus_{ev} G'')_e$

**Proof:** Proof is similar to Theorem 3.1.12.

**3.3. Subedge Operation**

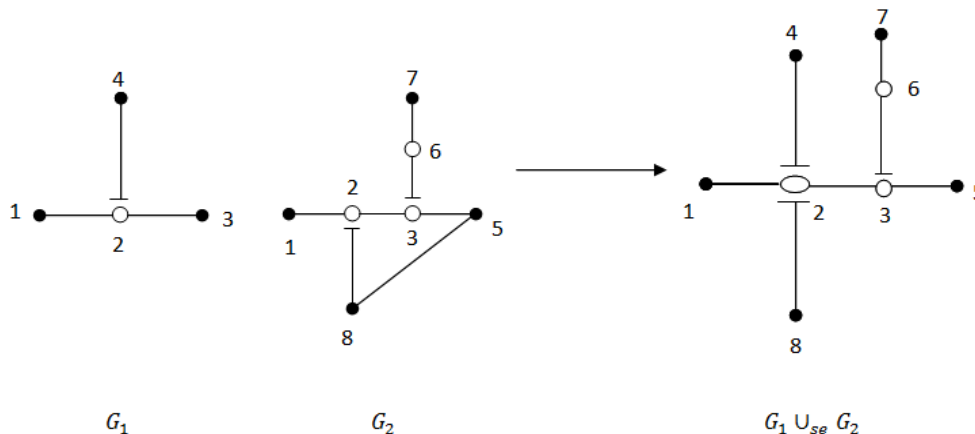
**Definition 3.3.1: Subedge Union of Semigraphs**

If  $G_1 = (V_1, X_1)$  and  $G_2 = (V_2, X_2)$  be two semigraphs then the subedge union of  $G_1$  and  $G_2$  denoted by  $G_1 \cup_{se} G_2$  is a semigraph with vertex set  $V_1 \cup V_2$  and edge set satisfying the following conditions:

- 1) If  $|E_{X_1} \cap E_{X_2}| \leq 1$  then  $E_{X_1}$  and  $E_{X_2} \in G_1 \cup_{se} G_2$  where  $E_{X_1} \in G_1$  and  $E_{X_2} \in G_2$ .
- 2) If  $|E_{X_1} \cap E_{X_2}| \geq 2$  such that no other subedge  $E_{X_i}, i \neq 1$  and  $i \neq 2$  contains common vertices of  $E_{X_1}$  and  $E_{X_2}$ , then  $E = E_{X_1} \cup E_{X_2} \in G_1 \cup_{se} G_2$  is an edge which is a permutation of vertices of  $V(E_{X_1}) \cup V(E_{X_2})$  retaining the vertex order of  $E_{X_1}$  and  $E_{X_2}$ .

Clearly, the subedge union of semigraphs is not unique.

**Example 3.3.2:**



**Figure 7.**

**Remarks 3.3.3:**

- i) For any semigraph  $G$  on  $n$  vertices,  $G \cup_{ss} G \cong_{\theta} G$ .
- ii) For any semigraph  $G$  on  $n$  vertices,  $G \cup_{ss} G^c \cong_{\theta} K_n$  where  $K_n$  is a complete semigraph containing both  $G$  and  $G^c$ .
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \cup_{ss} G_2 \cong_{\theta} G_2 \cup_{ss} G_1$ .
- iv) For any semigraphs  $G_1, G_2$  and  $G_3$ ,  $G_1 \cup_{ss} (G_2 \cup_{ss} G_3) \cong_{\theta} (G_1 \cup_{ss} G_2) \cup_{ss} G_3$
- v) The definition of subedge union can be extended to  $k$  number of graphs for  $k \geq 3$ .

**Theorem 3.3.4:** If  $G_a$  represent the adjacency graph of a semigraph  $G$ , then for any two semigraphs  $G'$  and  $G''$  we have  $G'_a \cup_{ss} G''_a \cong_{\theta} (G' \cup_{ss} G'')_a$ .

**Proof:** By definition 5.1, the vertex set of  $G'_a \cup_{ss} G''_a$  is same as the vertex set of  $(G' \cup_{ss} G'')_a$ . Let,  $E_{X_1}$  and  $E_{X_2}$  belong to  $E(G' \cup_{ss} G'')$ . This implies either  $E_{X_1} \in E(G')$  and  $E_{X_2} \in E(G'')$  or  $E_{X_1} \in E(G'')$  and  $E_{X_2} \in E(G')$ . Without loss of generality, let  $E_{X_1} \in E(G')$  and  $E_{X_2} \in E(G'')$  such that no other subedge contains common vertices of  $E_{X_1}$  and  $E_{X_2}$ . Then the edge  $E_{X_1}$  corresponds to a clique  $C'$  in  $G'_a$  and the edge  $E_{X_2}$  corresponds to a clique  $C''$  in  $G''_a$ . This implies that for each edge  $E = E_{X_1} \cup E_{X_2} \in E(G' \cup_{ss} G'')$  there corresponds a clique  $C = C' \cup C'' \in (G'_a \cup_{ss} G''_a)$ .

Now for the same edges  $E_{X_1}$  and  $E_{X_2} \in E(G' \cup_{ss} G'')$  there exists a clique  $C = C' \cup C'' \in (G'_a \cup_{ss} G''_a)$ . Hence for the edge  $E = E_{X_1} \cup E_{X_2}$  of  $G' \cup_{ss} G''$  there exists a clique  $C = C' \cup C'' \in (G'_a \cup_{ss} G''_a)$  and  $C = C' \cup C'' \in (G' \cup_{ss} G'')_a$ , with no vertices common to other subedges.

To show the existence of isomorphism between  $G'_a \cup_{ss} G''_a$  and  $(G' \cup_{ss} G'')_a$  we establish the following result. Let  $E = (E_1, E_2, \dots, E_k)$  be the set of edges of  $G' \cup_{ss} G''$  and  $C = (C_1, C_2, \dots, C_k)$  be the cliques in  $G'_a \cup_{ss} G''_a$ . Let  $U = \{u_1, u_2, \dots, u_i\}$  be the set of vertices which belongs to either  $E_{X_1}$  or  $E_{X_2}$  but does not belong to any other subedge of either  $G'$  or  $G''$ .

Let,  $f: (E \cup u_i) \rightarrow (C \cup u_i)$  be a function defined as  $f(E_i) = (E_{E_{X_1} \cup E_{X_2}}) \cup u_i$ .

Then, we have the following

- (i)  $|E_{X_i} \cup E_{X_j}| = |C_i \cup C_j|, i \neq j$
- (ii) If,  $E_i \neq E_j$  with  $|E_i \cap E_j| \geq 2$ , then  $f(E_i \cup E_j) = C_i \cup C_j$  with  $|C_i \cap C_j| \geq 2$ . For otherwise if  $E_i \neq E_j$  and  $f(E_i) = f(E_j)$  then  $|E_{E_{X_1} \cup E_{X_2}}| = |E| - 1$ , which is a contradiction to (i).

Therefore, by definition 2.14  $f$  is an  $e$ -isomorphism.

Hence,  $G'_a \cup_{ss} G''_a \cong_{\theta} (G' \cup_{ss} G'')_a$ .

**Definition 3.3.5: Subedge Intersection of Semigraphs**

Let  $G_1 = (V_1, X_1)$  and  $G_2 = (V_2, X_2)$  be two semigraphs then  $G_1 \cap_{ss} G_2$  is a semigraph with vertex set  $V_1 \cap V_2$  and edge set satisfying the following the conditions:

- 1) If  $|E_{X_1} \cap E_{X_2}| \leq 1$ , then  $E_{X_1}$  and  $E_{X_2} \in G_1 \cap_{ss} G_2$  where  $E_{X_1} \in G_1$  and  $E_{X_2} \in G_2$ .
- 2) If  $|E_{X_1} \cap E_{X_2}| \geq 2$  such that no other subedge  $E_{X_i}, i \neq 1$  and  $i \neq 2$  contains common vertices of  $E_{X_1}$  and  $E_{X_2}$ , then  $E = E_{X_1} \cap E_{X_2} \in G_1 \cap_{ss} G_2$  is an edge which is a permutation of vertices of  $V(E_{X_1}) \cap V(E_{X_2})$  retaining the vertex order of  $E_{X_1}$  and  $E_{X_2}$ .

Clearly the subedge intersection of semigraphs is not unique.

**Remarks 3.3.6:**

- i) For any semigraph  $G$ ,  $G \cap_{ss} G \cong_{\theta} G$ .
- ii) For any semigraph  $G$ ,  $G \cap_{ss} G^c$  is a null graph.
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \cap_{ss} G_2 \cong_{\theta} G_2 \cap_{ss} G_1$ .
- iv) For any semigraphs  $G_1, G_2$  and  $G_3$ ,  $G_1 \cap_{ss} (G_2 \cap_{ss} G_3) \cong_{\theta} (G_1 \cap_{ss} G_2) \cap_{ss} G_3$ .
- v) For any semigraphs  $G_1, G_2$  and  $G_3$ 
  - a)  $G_1 \cap_{ss} (G_2 \cap_{ss} G_3) \cong_{\theta} (G_1 \cap_{ss} G_2) \cap_{ss} (G_1 \cap_{ss} G_3)$
  - b)  $G_1 \cup_{ss} (G_2 \cap_{ss} G_3) \cong_{\theta} (G_1 \cup_{ss} G_2) \cap_{ss} (G_1 \cup_{ss} G_3)$

**Theorem 3.3.7:** If  $G_\alpha$  is the adjacency graph of a semigraph  $G$  then for any two semigraphs  $G'$  and  $G''$  we have  $G'_\alpha \cap_{se} G''_\alpha \cong (G' \cap_{se} G'')_\alpha$ .

**Proof:** Proof is similar to Theorem 3.1.8

**Definition 3.3.8: Subedge Decomposition**

A semigraph  $G$  on  $n$  vertices is said to have a subedge decomposition into two subsemigraphs  $G_1$  and  $G_2$  if  $G_1 \cup_{se} G_2 = G$  and  $G_1 \cap_{se} G_2 = \overline{K}_n$ .

**Definition 3.3.9: Ring Sum of Subedge**

The subedge ring sum of two semigraphs  $G_1$  and  $G_2$  denoted by  $G_1 \oplus_{se} G_2$  is defined as a semigraph with vertex set  $V_1 \cup V_2$  and the edges are the subedges present either in  $G_1$  or  $G_2$  but not in both.

**Remarks 3.3.10:**

- i) For any semigraph  $G$ ,  $G \oplus_{se} G = \overline{K}_n$ , is a null graph.
- ii) For any semigraph  $G$ ,  $G \oplus_{se} \overline{K}_n = G$ .
- iii) For any two semigraphs  $G_1$  and  $G_2$ ,  $G_1 \oplus_{se} G_2 = G_2 \oplus_{se} G_1$ .
- iv) For any semigraphs  $G_1$ ,  $G_2$  and  $G_3$ ,  $G_1 \oplus_{se} (G_2 \oplus_{se} G_3) = (G_1 \oplus_{se} G_2) \oplus_{se} G_3$ .

**Conclusion**

The concept of union, intersection, ring sum and decomposition is defined for semigraphs. The existence of isomorphism between a semigraph and its adjacency graph, consecutive adjacency graph, end-vertex graph is established. Relaxing the conditions imposed in all the definitions better generalizations are possible. The application of elementary operations to the p-Eulerian semigraph is also proved.

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