

## Seidel Equienergetic Graphs

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**Abstract.** The Seidel matrix  $S(G)$  of a graph  $G$  is the square matrix with diagonal entries zeroes and off diagonal entries are  $-1$  or  $1$  corresponding to the adjacency and non-adjacency. The Seidel energy  $SE(G)$  of  $G$  is defined as the sum of the absolute values of the eigenvalues of  $S(G)$ . Two graphs  $G_1$  and  $G_2$  are said to be Seidel equienergetic if  $SE(G_1) = SE(G_2)$ . We establish an expression for the characteristic polynomial of the Seidel matrix and for the Seidel energy of the join of regular graphs. Thereby construct Seidel non cospectral, Seidel equienergetic graphs on  $n$  vertices, for all  $n \geq 12$ .

### Introduction

Let  $G$  be a simple graph on  $n$  vertices and  $m$  edges with vertex set  $V(G) = \{v_1, v_2, \dots, v_n\}$ . The *Seidel matrix* of a graph  $G$  is an  $n \times n$  real symmetric matrix  $S(G) = [s_{ij}]$ , where  $s_{ij} = -1$  if the vertices  $v_i$  and  $v_j$  are adjacent,  $s_{ij} = 1$  if the vertices  $v_i$  and  $v_j$  are not adjacent and  $s_{ij} = 0$  if  $i = j$ . Obviously  $S(G) = J - I - 2A$ , where  $J$  is the matrix whose all entries are equal to 1,  $I$  is an identity matrix and  $A$  is the adjacency matrix of  $G$ . The *characteristic polynomial* of  $S(G)$  is defined as  $\phi(G : \sigma) = \det(\sigma I - S(G))$ . Let  $\sigma_1, \sigma_2, \dots, \sigma_n$  be the eigenvalues of  $S(G)$ .

The collection of the eigenvalues of the Seidel matrix of a graph is called the *Seidel spectra* of  $G$  [6]. Two non-isomorphic graphs are said to be *Seidel cospectral* if their Seidel matrices have same eigenvalues.

The *Seidel energy*  $SE(G)$  of a graph  $G$  is defined as [13]

$$SE(G) = \sum_{i=1}^n |\sigma_i|. \quad (1)$$

The Eq. (1) is in full analogy to the *ordinary graph energy* defined as the sum of the absolute values of the eigenvalues of the adjacency matrix of  $G$  [12]. For more details about the graph energy we refer [16]. Results on Seidel energy have been obtained in [11, 13, 18, 20].

The graphs  $G_1$  and  $G_2$  are said to be *Seidel equienergetic* if  $SE(G_1) = SE(G_2)$ . For obvious reason Seidel cospectral graphs are Seidel equienergetic. Several results on equienergetic graphs [2, 4, 5, 15, 24, 27, 28, 29, 30, 31], distance equienergetic graphs [14, 23, 26], Laplacian equienergetic graphs [8], equienergetic digraphs [17], skew equienergetic digraphs [1, 25], equienergetic sigraphs [3, 9, 10, 19] and equienergetic sidigraphs [21] were reported in the literature.

In [22], the Seidel energy of iterated line graphs of regular graphs was obtained and thus constructed the Seidel equienergetic graphs having different spectra. In this paper we obtain the characteristic polynomial of the Seidel matrix of the join of two regular graphs and the Seidel energy of join of regular graphs. Further we show that how to construct pairs of Seidel equienergetic graphs having different Seidel spectra, on  $n$  vertices for all  $n \geq 12$ .

**Seidel spectra and Seidel energy of join of graphs**

**Definition 1:** Let  $G_1$  be the graph with vertex set  $V_1$  and edge set  $E_1$  and let  $G_2$  be another graph with vertex set  $V_2$  and edge set  $E_2$ . The union of  $G_1$  and  $G_2$  is the graph  $G_1 \cup G_2$  whose vertex set is  $V_1 \cup V_2$  and edge set is  $E_1 \cup E_2$ .

**Definition 2 [7]:** The join of two graphs  $G_1$  and  $G_2$ , denoted by  $G_1 \nabla G_2$ , is a graph obtained from  $G_1 \cup G_2$  by joining each vertex of  $G_1$  to all vertices of  $G_2$ .

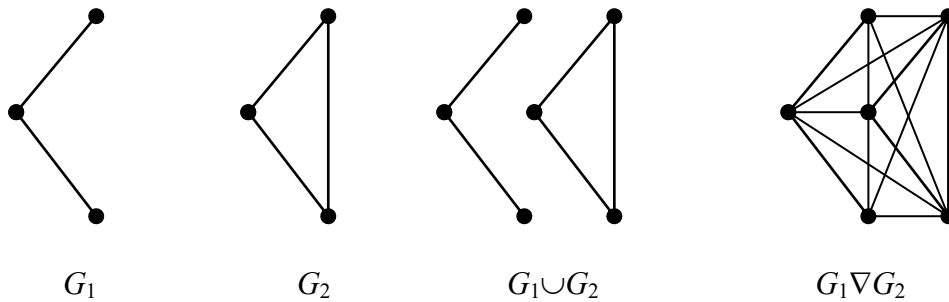


Figure 1

**Theorem 1:** Let  $G_i$  be an  $r_i$  - regular graph on  $n_i$  vertices,  $i = 1, 2$ . Then the characteristic polynomial of the Seidel matrix of  $G_1 \nabla G_2$  is

$$\phi(G_1 \nabla G_2 : \sigma) = \frac{[(\sigma - X)(\sigma - Y) - n_1 n_2]}{(\sigma - X)(\sigma - Y)} \phi(G_1 : \sigma) \phi(G_2 : \sigma), \tag{2}$$

where  $X = n_1 - 1 - 2r_1$  and  $Y = n_2 - 1 - 2r_2$ .

Proof:  $\phi(G_1 \nabla G_2 : \sigma) = \det(\sigma I - S(G_1 \nabla G_2))$

$$= \begin{vmatrix} \sigma I_{n_1} - S(G_1) & -J_{n_1 \times n_2} \\ -J_{n_2 \times n_1} & \sigma I_{n_2} - S(G_2) \end{vmatrix}, \tag{3}$$

where  $J$  is a matrix whose all entries are equal to one.

The determinant (3) can be written as

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & 1 & 1 & \cdots & 1 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & 1 & 1 & \cdots & 1 \\ \vdots & & \ddots & & & & \ddots & \\ -s_{n_1 1} & -s_{n_1 2} & \cdots & \sigma & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & \sigma & -s'_{12} & \cdots & -s'_{1n_2} \\ 1 & 1 & \cdots & 1 & -s'_{21} & \sigma & \cdots & -s'_{2n_2} \\ \vdots & & \ddots & & & & \ddots & \\ 1 & 1 & \cdots & 1 & -s'_{n_2 1} & -s'_{n_2 2} & \cdots & \sigma \end{vmatrix}, \tag{4}$$

where  $s_{ij}$  is the  $(i, j)$ -th entry in  $S(G_1)$ ,  $i, j = 1, 2, \dots, n_1$  and  $s'_{ij}$  is the  $(i, j)$ -th entry in  $S(G_2)$ ,  $i, j = 1, 2, \dots, n_2$ . Since  $G_i$  is an  $r_i$  -regular graph, every vertex of  $G_i$  is adjacent to  $r_i$  vertices and not adjacent to  $n - 1 - r_i$  vertices,  $i = 1, 2$ . Therefore

$$\sum_{j=1}^{n_1} s_{ij} = n_1 - 1 - 2r_1 \text{ for } i = 1, 2, \dots, n_1 \tag{5}$$

and

$$\sum_{j=1}^{n_2} s'_{ij} = n_2 - 1 - 2r_2 \text{ for } i = 1, 2, \dots, n_2. \tag{6}$$

We now perform a number of transformations that leave the value of the determinant (4) unchanged.

Subtract the row  $(n_1 + 1)$  from the rows  $(n_1 + 2), (n_1 + 3), \dots, (n_1 + n_2)$  of (4) to obtain (7):

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & 1 & 1 & \cdots & 1 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & 1 & 1 & \cdots & 1 \\ \vdots & & \vdots & & & & \vdots & \\ -s_{n_11} & -s_{n_12} & \cdots & \sigma & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & \sigma & -s'_{12} & \cdots & -s'_{1n_2} \\ 0 & 0 & \cdots & 0 & -s'_{21}-\sigma & \sigma+s'_{12} & \cdots & -s'_{2n_2}+s'_{1n_2} \\ \vdots & & \vdots & & & & \vdots & \\ 0 & 0 & \cdots & 0 & -s'_{n_21}-\sigma & -s'_{n_22}+s'_{12} & \cdots & \sigma+s'_{1n_2} \end{vmatrix}. \quad (7)$$

Adding the columns  $(n_1 + 2), (n_1 + 3), \dots, (n_1 + n_2)$  to the column  $(n_1 + 1)$  of (7) and using Eq. (6) we arrive at the determinant (8), where  $Y = n_2 - 1 - 2r_2$ :

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & n_2 & 1 & \cdots & 1 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & n_2 & 1 & \cdots & 1 \\ \vdots & & \vdots & & & & \vdots & \\ -s_{n_11} & -s_{n_12} & \cdots & \sigma & n_2 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & \sigma - Y & -s'_{12} & \cdots & -s'_{1n_2} \\ 0 & 0 & \cdots & 0 & 0 & \sigma + s'_{12} & \cdots & -s'_{2n_2} + s'_{1n_2} \\ \vdots & & \vdots & & & & \vdots & \\ 0 & 0 & \cdots & 0 & 0 & -s'_{n_22} + s'_{12} & \cdots & \sigma + s'_{1n_2} \end{vmatrix} \quad (8)$$

which evidently is equal to (9):

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & n_2 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & n_2 \\ \vdots & & \vdots & & \\ -s_{n_11} & -s_{n_12} & \cdots & \sigma & n_2 \\ 1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix} |B| \quad (9)$$

where

$$|B| = \begin{vmatrix} \sigma + s'_{12} & -s'_{23} + s'_{13} & \cdots & -s'_{2n_2} + s'_{1n_2} \\ -s'_{32} + s'_{12} & \sigma + s'_{13} & \cdots & -s'_{3n_2} + s'_{1n_2} \\ \vdots & & \vdots & \\ -s'_{n_22} + s'_{12} & -s'_{n_23} + s'_{13} & \cdots & \sigma + s'_{1n_2} \end{vmatrix}. \quad (10)$$

In (9) the first determinant is of order  $(n_1 + 1)$ . Subtract the first row from the rows  $2, 3, \dots, n_1$ , to obtain (11):

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & n_2 \\ -s_{21} - \sigma & \sigma + s_{12} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ \vdots & & \vdots & & \\ -s_{n_11} - \sigma & -s_{n_12} + s_{12} & \cdots & \sigma + s_{1n_1} & 0 \\ 1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix} |B|. \quad (11)$$

Adding columns 2, 3, ...,  $n_1$  to the first column of (11) and using Eq. (5) we get (12), where  $X = n_1 - 1 - 2r_1$ :

$$\begin{vmatrix} \sigma - X & -s_{12} & \cdots & -s_{1n_1} & n_2 \\ 0 & \sigma + s_{12} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -s_{n_1 2} + s_{12} & \cdots & \sigma + s_{1n_1} & 0 \\ n_1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix} |B|. \tag{12}$$

Expand it along the first column to obtain (13):

$$\{(\sigma - X)\Delta_1 + (-1)^{n_1} n_1 \Delta_2\} |B| \tag{13}$$

where

$$\Delta_1 = \begin{vmatrix} \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} & 0 \\ 1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix}$$

and

$$\Delta_2 = \begin{vmatrix} -s_{12} & -s_{13} & \cdots & -s_{1n_1} & n_2 \\ \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} & 0 \end{vmatrix}.$$

The expression (13) can be rewritten as

$$\{(\sigma - X)(\sigma - Y) |A| + (-1)^{n_1} n_1 (-1)^{n_1+1} n_2 |A| \Delta_2\} |B| = \{(\sigma - X)(\sigma - Y) - n_1 n_2\} |A| |B|, \tag{14}$$

where

$$|A| = \begin{vmatrix} \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} \\ -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} \\ \vdots & \vdots & \vdots & \vdots \\ -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} \end{vmatrix}. \tag{15}$$

The determinant (15) can be written as

$$|A| = \frac{1}{(\sigma - X)} \begin{vmatrix} \sigma - X & -s_{12} & -s_{13} & \cdots & -s_{1n_1} \\ 0 & \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} \\ 0 & -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} \end{vmatrix}. \tag{16}$$

From Eq. (5) the sum of the  $i$ -th row in (16) is  $\sigma + s_{i1}$  for  $i = 2, 3, \dots, n_1$ . Therefore, by subtracting the columns 2, 3, ...,  $n_1$  of (16) from the first column, we obtain (17):

$$|A| = \frac{1}{(\sigma - X)} \begin{vmatrix} \sigma - X & -s_{12} & -s_{13} & \cdots & -s_{1n_1} \\ -\sigma - s_{21} & \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} \\ -\sigma - s_{31} & -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\sigma - s_{n_1 1} & -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} \end{vmatrix}. \tag{17}$$

Add the first row of (17) to the rows 2, 3, ...,  $n_1$  to obtain (18):

$$|A| = \frac{1}{(\sigma - X)} \begin{vmatrix} \sigma & -s_{12} & -s_{13} & \cdots & -s_{1n_1} \\ -s_{21} & \sigma & -s_{23} & \cdots & -s_{2n_1} \\ -s_{31} & -s_{32} & \sigma & \cdots & -s_{3n_1} \\ \vdots & & \vdots & \ddots & \\ -s_{n_11} & -s_{n_12} & -s_{n_13} & \cdots & \sigma \end{vmatrix} \\ = \frac{1}{(\sigma - X)} \phi(G_1 : \sigma). \quad (18)$$

In a similar manner we can show that from (10) follows

$$|B| = \frac{1}{(\sigma - Y)} \phi(G_2 : \sigma). \quad (19)$$

Substituting (18) and (19) back into (14) gives Eq. (2).

Let  $U_1$  and  $U_2 = V(G) \setminus U_1$  be the partitioned sets of the vertex set  $V(G)$  of a graph  $G$ . Let  $G'$  be the graph obtained from  $G$  by deleting all edges between  $U_1$  and  $U_2$  and inserting all edges between  $U_1$  and  $U_2$  that were not present in  $G$ . Then  $G'$  and  $G$  are said to be *Seidel switching* with respect to  $U_1$ . If  $G'$  and  $G$  are Seidel switching then  $S(G')$  and  $S(G)$  are similar and therefore  $G'$  and  $G$  have same Seidel eigenvalues and equal Seidel energy [13]. Thus for any two graphs  $G_1$  and  $G_2$ , the Seidel matrices  $S(G_1 \nabla G_2)$  and  $S(G_1 \cup G_2)$  are similar and therefore have same characteristic polynomials and  $SE(G_1 \nabla G_2) = SE(G_1 \cup G_2)$ .

**Theorem 2:** Let  $G_i$  be an  $r_i$ -regular graph on  $n_i$  vertices,  $i = 1, 2$ . Then

$$SE(G_1 \nabla G_2) = SE(G_1) + SE(G_2) - (X + Y) + \sqrt{(X - Y)^2 + 4n_1n_2}$$

where  $X = n_1 - 1 - 2r_1$  and  $Y = n_2 - 1 - 2r_2$ .

**Proof:** From Theorem 1,

$$\phi(G_1 \nabla G_2 : \sigma) = \frac{[(\sigma - X)(\sigma - Y) - n_1n_2]}{(\sigma - X)(\sigma - Y)} \phi(G_1 : \sigma) \phi(G_2 : \sigma),$$

which gives that

$$(\sigma - X)(\sigma - Y) \phi(G_1 \nabla G_2 : \sigma) = [(\sigma - X)(\sigma - Y) - n_1n_2] \phi(G_1 : \sigma) \phi(G_2 : \sigma).$$

Let  $P_1(\sigma) = (\sigma - X)(\sigma - Y) \phi(G_1 \nabla G_2 : \sigma)$  and  $P_2(\sigma) = [(\sigma - X)(\sigma - Y) - n_1n_2] \phi(G_1 : \sigma) \phi(G_2 : \sigma)$ .

The roots of the equation  $P_1(\sigma) = 0$  are  $X, Y$  and the eigenvalues of  $S(G_1 \nabla G_2)$ . Therefore the sum of the absolute values of the roots of  $P_1(\sigma) = 0$  is

$$X + Y + SE(G_1 \nabla G_2). \quad (20)$$

The roots of  $P_2(\sigma) = 0$  are the eigenvalues of  $S(G_1)$ , eigenvalues of  $S(G_2)$  and

$$\frac{1}{2}(X + Y) \pm \sqrt{(X + Y)^2 - 4(XY - n_1n_2)}.$$

Therefore the sum of the absolute values of the roots of  $P_2(\sigma) = 0$  is

$$SE(G_1) + SE(G_2) + \left| \frac{1}{2}(X + Y) + \sqrt{(X + Y)^2 - 4(XY - n_1n_2)} \right| \\ + \left| \frac{1}{2}(X + Y) - \sqrt{(X + Y)^2 - 4(XY - n_1n_2)} \right|. \quad (21)$$

Since  $XY = (n_1 - 1 - 2r_1)(n_2 - 1 - 2r_2) < n_1n_2$ , Eq. (21) reduces to

$$SE(G_1) + SE(G_2) + \sqrt{(X + Y)^2 - 4(XY - n_1n_2)} = SE(G_1) + SE(G_2) + \sqrt{(X - Y)^2 + n_1n_2}. \quad (22)$$

Since  $P_1(\sigma) = P_2(\sigma)$ , equating Eqs. (20) and (22) we get  
 $SE(G_1 \nabla G_2) = SE(G_1) + SE(G_2) - (X + Y) + \sqrt{(X - Y)^2 + 4n_1n_2}$ .

Let  $\overline{K_n}$  be the totally disconnected graph on  $n$  vertices and  $K_{p,q}$  be the complete bipartite graph on  $p+q$  vertices.

**Corollary 2.1:**  $SE(K_{p,q}) = SE(\overline{K_p} \nabla \overline{K_q}) = 2(p + q - 1)$ .

**Corollary 2.2:** If  $H_1$  and  $H_2$  are Seidel non cospectral, Seidel equienergetic regular graphs on  $n$  vertices and of same degree, then for any regular graph  $G$ ,  $SE(H_1 \nabla G) = SE(H_2 \nabla G)$ .

**Construction of Seidel equienergetic graphs**

Consider the graphs  $H_a$  and  $H_b$  as shown in Fig. 2.

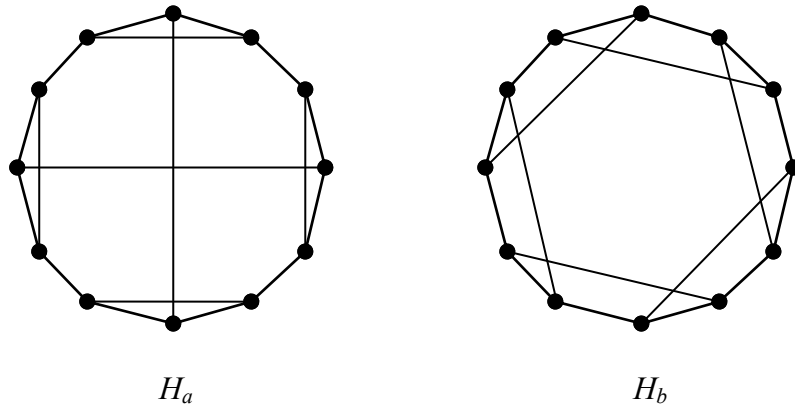


Figure 2

By direct computation,

$$\phi(H_a : \sigma) = (\sigma - 5)(\sigma - 3)^3(\sigma - 1)^3(\sigma + 1)^2(\sigma + 5)^3 \tag{23}$$

and

$$\phi(H_b : \sigma) = (\sigma - 5)^2(\sigma - 3)^2(\sigma - 1)(\sigma + 1)^4(\sigma + 3)(\sigma + 5)^2. \tag{24}$$

Both  $H_a$  and  $H_b$  are regular graphs on 12 vertices and of degree 3. And  $SE(H_a) = SE(H_b) = 34$ .

Let  $H$  be any  $r$ -regular graph on  $p \geq 1$  vertices. Then by Theorem 2,

$$SE(H_a \nabla H) = SE(H_b \nabla H) = SE(H) + 30 - p + 2r + \sqrt{(6 - p + 2r)^2 + 48p}.$$

Thus  $H_a \nabla H$  and  $H_b \nabla H$  are Seidel equienergetic. By Eqs. (23) and (24),  $H_a$  and  $H_b$  are Seidel non cospectral, so by Theorem 1,  $H_a \nabla H$  and  $H_b \nabla H$  are also Seidel non cospectral. Further  $H_a \nabla H$  and  $H_b \nabla H$  possesses equal number of vertices  $n = 12 + p, p = 1, 2, \dots$

Also for  $n = 12$ , the Seidel equienergeticity is directly verified from Eqs. (23) and (24).

From above construction, we state the following Theorem.

**Theorem 3:** There exist pairs of Seidel non cospectral, Seidel equienergetic graphs on  $n$  vertices, for all  $n \geq 12$ .

Let  $K_p$  be the complete graph on  $p$  vertices. It is regular of degree  $p - 1$ . The Seidel matrix of  $K_p$  is the negation of its adjacency matrix. Therefore  $SE(K_p) = 2(p - 1)$  [7]. Using this in Theorem 2 we have following result.

**Theorem 4:** If  $H_a$  and  $H_b$  are the graphs as shown in Fig. 2, then for  $p \geq 1$ ,

$$SE(H_a \nabla K_p) = SE(H_b \nabla K_p) = 3p + 26 + \sqrt{p^2 + 56p + 16}.$$

## Conclusion

From Corollary 2.2, the construction of Seidel non cospectral, Seidel equienergetic graphs on equal number of vertices can be done. In particular from Theorem 3 and Theorem 4, it is easy to construct a pair of Seidel non cospectral, Seidel equienergetic  $n$ -vertex graphs for all  $n \geq 12$ .

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