Bulletin of Mathematical Sciences and Applications
ISSN: 2278-9634, Vol. 16, pp 62-69
doi:10.18052/www.scipress.com/BMSA.16.62

Submitted: 2016-03-24
Revised: 2016-03-27
Accepted: 2016-05-13
Online: 2016-08-04

Seidel Equienergetic Graphs

Harishchandra S. Ramane[1, a *], Mahadevappa M. Gundloor[1,b] and Sunilkumar M. Hosamani[2,c]

[1]Department of Mathematics, Karnatak University, Dharwad - 580003, India

[2]Department of Mathematics, Rani Channamma University, Belgaum - 591156, India

[a]hsramane@yahoo.com, [b]madeshgundloor@gmail.com, [c]sunilkumar.rcu@gmail.com

* Corresponding author: Harishchandra S. Ramane

Keywords: Seidel eigenvalues, join of graphs, Seidel energy.

Abstract. The Seidel matrix $S(G)$ of a graph G is the square matrix with diagonal entries zeroes and off diagonal entries are – 1 or 1 corresponding to the adjacency and non-adjacency. The Seidel energy $SE(G)$ of G is defined as the sum of the absolute values of the eigenvalues of $S(G)$. Two graphs G_1 and G_2 are said to be Seidel equienergetic if $SE(G_1) = SE(G_2)$. We establish an expression for the characteristic polynomial of the Seidel matrix and for the Seidel energy of the join of regular graphs. Thereby construct Seidel non cospectral, Seidel equienergetic graphs on n vertices, for all $n \geq 12$.

Introduction

Let G be a simple graph on n vertices and m edges with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. The *Seidel matrix* of a graph G is an $n \times n$ real symmetric matrix $S(G) = [s_{ij}]$, where $s_{ij} = -1$ if the vertices v_i and v_j are adjacent, $s_{ij} = 1$ if the vertices v_i and v_j are not adjacent and $s_{ij} = 0$ if $i = j$. Obviously $S(G) = J - I - 2A$, where J is the matrix whose all entries are equal to 1, I is an identity matrix and A is the adjacency matrix of G. The *characteristic polynomial* of $S(G)$ is defined as $\phi(G : \sigma) = \det(\sigma I - S(G))$. Let $\sigma_1, \sigma_2, \ldots, \sigma_n$ be the eigenvalues of $S(G)$.

The collection of the eigenvalues of the Seidel matrix of a graph is called the *Seidel spectra* of G [6]. Two non-isomorphic graphs are said to be *Seidel cospectral* if their Seidel matrices have same eigenvalues.

The *Seidel energy* $SE(G)$ of a graph G is defined as [13]

$$SE(G) = \sum_{i=1}^{n} |\sigma_i| . \tag{1}$$

The Eq. (1) is in full analogy to the *ordinary graph energy* defined as the sum of the absolute values of the eigenvalues of the adjacency matrix of G [12]. For more details about the graph energy we refer [16]. Results on Seidel energy have been obtained in [11, 13, 18, 20].

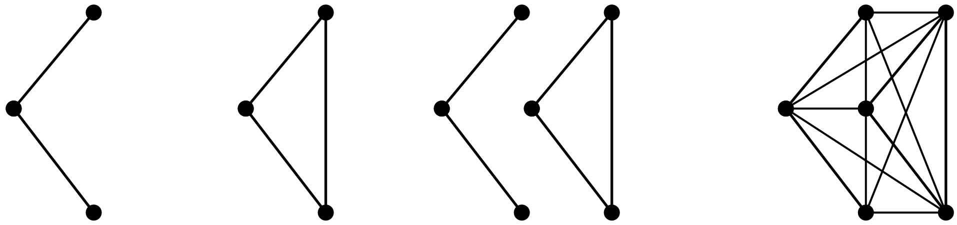
The graphs G_1 and G_2 are said to be *Seidel equienergetic* if $SE(G_1) = SE(G_2)$. For obvious reason Seidel cospectral graphs are Seidel equienergetic. Several results on equienergetic graphs [2, 4, 5, 15, 24, 27, 28, 29, 30, 31], distance equienergetic graphs [14, 23, 26], Laplacian equienergetic graphs [8], equienergetic digraphs [17], skew equienergetic digraphs [1, 25], equienergetic sigraphs [3, 9, 10, 19] and equienergetic sidigraphs [21] were reported in the literature.

In [22], the Seidel energy of iterated line graphs of regular graphs was obtained and thus constructed the Seidel equienergetic graphs having different spectra. In this paper we obtain the characteristic polynomial of the Seidel matrix of the join of two regular graphs and the Seidel energy of join of regular graphs. Further we show that how to construct pairs of Seidel equienergetic graphs having different Seidel spectra, on n vertices for all $n \geq 12$.

Seidel spectra and Seidel energy of join of graphs

Definition 1: Let G_1 be the graph with vertex set V_1 and edge set E_1 and let G_2 be another graph with vertex set V_2 and edge set E_2. The union of G_1 and G_2 is the graph $G_1 \cup G_2$ whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2$.

Definition 2 [7]**:** The join of two graphs G_1 and G_2, denoted by $G_1 \nabla G_2$, is a graph obtained from $G_1 \cup G_2$ by joining each vertex of G_1 to all vertices of G_2.



G_1 G_2 $G_1 \cup G_2$ $G_1 \nabla G_2$

Figure 1

Theorem 1: Let G_i be an r_i - regular graph on n_i vertices, $i = 1, 2$. Then the characteristic polynomial of the Seidel matrix of $G_1 \nabla G_2$ is

$$\phi(G_1 \nabla G_2 : \sigma) = \frac{[(\sigma - X)(\sigma - Y) - n_1 n_2]}{(\sigma - X)(\sigma - Y)} \phi(G_1 : \sigma)\phi(G_2 : \sigma), \tag{2}$$

where $X = n_1 - 1 - 2r_1$ and $Y = n_2 - 1 - 2r_2$.

Proof: $\phi(G_1 \nabla G_2 : \sigma) = \det(\sigma I - S(G_1 \nabla G_2))$

$$= \begin{vmatrix} \sigma I_{n_1} - S(G_1) & -J_{n_1 \times n_2} \\ -J_{n_2 \times n_1} & \sigma I_{n_2} - S(G_2) \end{vmatrix}, \tag{3}$$

where J is a matrix whose all entries are equal to one.

The determinant (3) can be written as

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & 1 & 1 & \cdots & 1 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & 1 & 1 & \cdots & 1 \\ \vdots & & \vdots & & & & \vdots & \\ -s_{n_1 1} & -s_{n_1 2} & \cdots & \sigma & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & \sigma & -s'_{12} & \cdots & -s'_{1n_2} \\ 1 & 1 & \cdots & 1 & -s'_{21} & \sigma & \cdots & -s'_{2n_2} \\ \vdots & & \vdots & & & & \vdots & \\ 1 & 1 & \cdots & 1 & -s'_{n_2 1} & -s'_{n_2 2} & \cdots & \sigma \end{vmatrix}, \tag{4}$$

where s_{ij} is the (i, j)-th entry in $S(G_1)$, $i, j = 1, 2, \ldots, n_1$ and s'_{ij} is the (i, j)-th entry in $S(G_2)$, $i, j = 1, 2, \ldots, n_2$. Since G_i is an r_i -regular graph, every vertex of G_i is adjacent to r_i vertices and not adjacent to $n - 1 - r_i$ vertices, $i = 1, 2$. Therefore

$$\sum_{j=1}^{n_1} s_{ij} = n_1 - 1 - 2r_1 \text{ for } i = 1, 2, \ldots, n_1 \tag{5}$$

and

$$\sum_{j=1}^{n_2} s'_{ij} = n_2 - 1 - 2r_2 \text{ for } i = 1, 2, \ldots, n_2. \tag{6}$$

We now perform a number of transformations that leave the value of the determinant (4) unchanged.

Subtract the row ($n_1 + 1$) from the rows ($n_1 + 2$), ($n_1 + 3$), … , ($n_1 + n_2$) of (4) to obtain (7):

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & 1 & 1 & \cdots & 1 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & 1 & 1 & \cdots & 1 \\ \vdots & & \vdots & & & & \vdots & \\ -s_{n_1 1} & -s_{n_1 2} & \cdots & \sigma & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & \sigma & -s'_{12} & \cdots & -s'_{1n_2} \\ 0 & 0 & \cdots & 0 & -s'_{21}-\sigma & \sigma + s'_{12} & \cdots & -s'_{2n_2} + s'_{1n_2} \\ \vdots & & \vdots & & & & \vdots & \\ 0 & 0 & \cdots & 0 & -s'_{n_2 1}-\sigma & -s'_{n_2 2}+s'_{12} & \cdots & \sigma + s'_{1n_2} \end{vmatrix} . \tag{7}$$

Adding the columns ($n_1 + 2$), ($n_1 + 3$), … , ($n_1 + n_2$) to the column ($n_1 + 1$) of (7) and using Eq. (6) we arrive at the determinant (8), where $Y = n_2 - 1 - 2r_2$:

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & n_2 & 1 & \cdots & 1 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & n_2 & 1 & \cdots & 1 \\ \vdots & & \vdots & & & & \vdots & \\ -s_{n_1 1} & -s_{n_1 2} & \cdots & \sigma & n_2 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & \sigma - Y & -s'_{12} & \cdots & -s'_{1n_2} \\ 0 & 0 & \cdots & 0 & 0 & \sigma + s'_{12} & \cdots & -s'_{2n_2} + s'_{1n_2} \\ \vdots & & \vdots & & & & \vdots & \\ 0 & 0 & \cdots & 0 & 0 & -s'_{n_2 2}+s'_{12} & \cdots & \sigma + s'_{1n_2} \end{vmatrix} \tag{8}$$

which evidently is equal to (9):

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & n_2 \\ -s_{21} & \sigma & \cdots & -s_{2n_1} & n_2 \\ \vdots & & \vdots & & \\ -s_{n_1 1} & -s_{n_1 2} & \cdots & \sigma & n_2 \\ 1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix} |B| \tag{9}$$

where

$$|B| = \begin{vmatrix} \sigma + s'_{12} & -s'_{23}+s'_{13} & \cdots & -s'_{2n_2} + s'_{1n_2} \\ -s'_{32}+s'_{12} & \sigma + s'_{13} & \cdots & -s'_{3n_2} + s'_{1n_2} \\ \vdots & & \vdots & \\ -s'_{n_2 2}+s'_{12} & -s'_{n_2 3}+s'_{13} & \cdots & \sigma + s'_{1n_2} \end{vmatrix} . \tag{10}$$

In (9) the first determinant is of order ($n_1 + 1$). Subtract the first row from the rows 2, 3, … , n_1, to obtain (11):

$$\begin{vmatrix} \sigma & -s_{12} & \cdots & -s_{1n_1} & n_2 \\ -s_{21}-\sigma & \sigma + s_{12} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ \vdots & & \vdots & & \\ -s_{n_1 1}-\sigma & -s_{n_1 2} + s_{12} & \cdots & \sigma + s_{1n_1} & 0 \\ 1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix} |B| . \tag{11}$$

Adding columns 2, 3, … , n_1 to the first column of (11) and using Eq. (5) we get (12), where $X = n_1 - 1 - 2r_1$:

$$\begin{vmatrix} \sigma - X & -s_{12} & \cdots & -s_{1n_1} & n_2 \\ 0 & \sigma + s_{12} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ \vdots & \vdots & & & \\ 0 & -s_{n_1 2} + s_{12} & \cdots & \sigma + s_{1n_1} & 0 \\ n_1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix} |B| . \qquad (12)$$

Expand it along the first column to obtain (13):

$$\{(\sigma - X)\Delta_1 + (-1)^{n_1} n_1 \Delta_2\} \, |B| \qquad (13)$$

where

$$\Delta_1 = \begin{vmatrix} \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} & 0 \\ \vdots & & \vdots & & \\ -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} & 0 \\ 1 & 1 & \cdots & 1 & \sigma - Y \end{vmatrix}$$

and

$$\Delta_2 = \begin{vmatrix} -s_{12} & -s_{13} & \cdots & -s_{1n_1} & n_2 \\ \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} & 0 \\ -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} & 0 \\ \vdots & & \vdots & & \\ -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} & 0 \end{vmatrix} .$$

The expression (13) can be rewritten as

$$\{(\sigma - X)(\sigma - Y)\,|A| + (-1)^{n_1} n_1 (-1)^{n_1+1} n_2 \,|A|\, \Delta_2\} \, |B| = \{(\sigma - X)(\sigma - Y) - n_1 n_2\} |A| \, |B| , \qquad (14)$$

where

$$|A| = \begin{vmatrix} \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} \\ -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} \\ \vdots & & \vdots & \\ -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} \end{vmatrix} . \qquad (15)$$

The determinant (15) can be written as

$$|A| = \frac{1}{(\sigma - X)} \begin{vmatrix} \sigma - X & -s_{12} & -s_{13} & \cdots & -s_{1n_1} \\ 0 & \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} \\ 0 & -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} \\ \vdots & & \vdots & \vdots & \\ 0 & -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} \end{vmatrix} . \qquad (16)$$

From Eq. (5) the sum of the i-th row in (16) is $\sigma + s_{i1}$ for $i = 2, 3, \ldots, n_1$. Therefore, by subtracting the columns 2, 3, … , n_1 of (16) from the first column, we obtain (17):

$$|A| = \frac{1}{(\sigma - X)} \begin{vmatrix} \sigma - X & -s_{12} & -s_{13} & \cdots & -s_{1n_1} \\ -\sigma - s_{21} & \sigma + s_{12} & -s_{23} + s_{13} & \cdots & -s_{2n_1} + s_{1n_1} \\ -\sigma - s_{31} & -s_{32} + s_{12} & \sigma + s_{13} & \cdots & -s_{3n_1} + s_{1n_1} \\ \vdots & & \vdots & \vdots & \\ -\sigma - s_{n_1 1} & -s_{n_1 2} + s_{12} & -s_{n_1 3} + s_{13} & \cdots & \sigma + s_{1n_1} \end{vmatrix} . \qquad (17)$$

Add the first row of (17) to the rows 2, 3, … , n_1 to obtain (18):

$$|A| = \frac{1}{(\sigma - X)} \begin{vmatrix} \sigma & -s_{12} & -s_{13} & \cdots & -s_{1n_1} \\ -s_{21} & \sigma & -s_{23} & \cdots & -s_{2n_1} \\ -s_{31} & -s_{32} & \sigma & \cdots & -s_{3n_1} \\ \vdots & & \vdots & \vdots & \\ -s_{n_1 1} & -s_{n_1 2} & -s_{n_1 3} & \cdots & \sigma \end{vmatrix}$$

$$= \frac{1}{(\sigma - X)} \phi(G_1 : \sigma) \ . \tag{18}$$

In a similar manner we can show that from (10) follows

$$|B| = \frac{1}{(\sigma - Y)} \phi(G_2 : \sigma) \ . \tag{19}$$

Substituting (18) and (19) back into (14) gives Eq. (2).

Let U_1 and $U_2 = V(G) \setminus U_1$ be the partitioned sets of the vertex set $V(G)$ of a graph G. Let G' be the graph obtained from G by deleting all edges between U_1 and U_2 and inserting all edges between U_1 and U_2 that were not present in G. Then G' and G are said to be *Seidel switching* with respect to U_1. If G' and G are Seidel switching then $S(G')$ and $S(G)$ are similar and therefore G' and G have same Seidel eigenvalues and equal Seidel energy [13]. Thus for any two graphs G_1 and G_2, the Seidel matrices $S(G_1 \nabla G_2)$ and $S(G_1 \cup G_2)$ are similar and therefore have same characteristic polynomials and $SE(G_1 \nabla G_2) = SE(G_1 \cup G_2)$.

Theorem 2: Let G_i be an r_i-regular graph on n_i vertices, $i = 1, 2$. Then

$$SE(G_1 \nabla G_2) = SE(G_1) + SE(G_2) - (X + Y) + \sqrt{(X - Y)^2 + 4n_1 n_2}$$

where $X = n_1 - 1 - 2r_1$ and $Y = n_2 - 1 - 2r_2$.

Proof: From Theorem 1,

$$\phi(G_1 \nabla G_2 : \sigma) = \frac{[(\sigma - X)(\sigma - Y) - n_1 n_2]}{(\sigma - X)(\sigma - Y)} \phi(G_1 : \sigma) \phi(G_2 : \sigma) ,$$

which gives that

$$(\sigma - X)(\sigma - Y)\phi(G_1 \nabla G_2 : \sigma) = [(\sigma - X)(\sigma - Y) - n_1 n_2] \phi(G_1 : \sigma) \phi(G_2 : \sigma) .$$

Let $P_1(\sigma) = (\sigma - X)(\sigma - Y)\phi(G_1 \nabla G_2 : \sigma)$ and $P_2(\sigma) = [(\sigma - X)(\sigma - Y) - n_1 n_2]\phi(G_1 : \sigma)\phi(G_2 : \sigma)$.

The roots of the equation $P_1(\sigma) = 0$ are X, Y and the eigenvalues of $S(G_1 \nabla G_2)$. Therefore the sum of the absolute values of the roots of $P_1(\sigma) = 0$ is

$$X + Y + SE(G_1 \nabla G_2) . \tag{20}$$

The roots of $P_2(\sigma) = 0$ are the eigenvalues of $S(G_1)$, eigenvalues of $S(G_2)$ and

$$\frac{1}{2}(X + Y) \pm \sqrt{(X + Y)^2 - 4(XY - n_1 n_2)} .$$

Therefore the sum of the absolute values of the roots of $P_2(\sigma) = 0$ is

$$SE(G_1) + SE(G_2) + \left| \frac{1}{2}(X + Y) + \sqrt{(X + Y)^2 - 4(XY - n_1 n_2)} \right|$$
$$+ \left| \frac{1}{2}(X + Y) - \sqrt{(X + Y)^2 - 4(XY - n_1 n_2)} \right| . \tag{21}$$

Since $XY = (n_1 - 1 - 2r_1)(n_2 - 1 - 2r_2) < n_1 n_2$, Eq. (21) reduces to

$$SE(G_1) + SE(G_2) + \sqrt{(X + Y)^2 - 4(XY - n_1 n_2)} = SE(G_1) + SE(G_2) + \sqrt{(X - Y)^2 + n_1 n_2} . \tag{22}$$

Since $P_1(\sigma) = P_2(\sigma)$, equating Eqs. (20) and (22) we get
$$SE(G_1 \nabla G_2) = SE(G_1) + SE(G_2) - (X+Y) + \sqrt{(X-Y)^2 + 4n_1 n_2}\ .$$

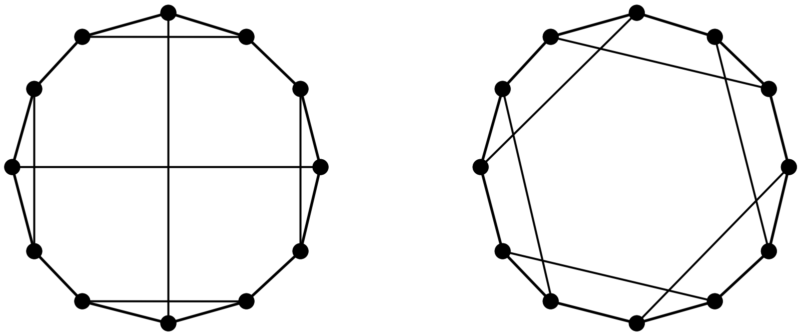
Let $\overline{K_n}$ be the totally disconnected graph on n vertices and $K_{p,q}$ be the complete bipartite graph on $p+q$ vertices.

Corollary 2.1: $SE(K_{p,q}) = SE(\overline{K_p} \nabla \overline{K_q}) = 2(p+q-1)$.

Corollary 2.2: If H_1 and H_2 are Seidel non cospectral, Seidel equienergetic regular graphs on n vertices and of same degree, then for any regular graph G, $SE(H_1 \nabla G) = SE(H_2 \nabla G)$.

Construction of Seidel equienergetic graphs

Consider the graphs H_a and H_b as shown in Fig. 2.



H_a H_b

Figure 2

By direct computation,
$$\phi(H_a : \sigma) = (\sigma-5)(\sigma-3)^3(\sigma-1)^3(\sigma+1)^2(\sigma+5)^3 \tag{23}$$
and
$$\phi(H_b : \sigma) = (\sigma-5)^2(\sigma-3)^2(\sigma-1)(\sigma+1)^4(\sigma+3)(\sigma+5)^2 . \tag{24}$$
Both H_a and H_b are regular graphs on 12 vertices and of degree 3. And $SE(H_a) = SE(H_b) = 34$. Let H be any r-regular graph on $p \geq 1$ vertices. Then by Theorem 2,
$$SE(H_a \nabla H) = SE(H_b \nabla H) = SE(H) + 30 - p + 2r + \sqrt{(6-p+2r)^2 + 48p}\ .$$
Thus $H_a \nabla H$ and $H_b \nabla H$ are Seidel equienergetic. By Eqs. (23) and (24), H_a and H_b are Seidel non cospectral, so by Theorem 1, $H_a \nabla H$ and $H_b \nabla H$ are also Seidel non cospectral. Further $H_a \nabla H$ and $H_b \nabla H$ possesses equal number of vertices $n = 12 + p$, $p = 1, 2, \ldots$.

Also for $n = 12$, the Seidel equienergicity is directly verified from Eqs. (23) and (24).
From above construction, we state the following Theorem.

Theorem 3: There exist pairs of Seidel non cospectral, Seidel equienergetic graphs on n vertices, for all $n \geq 12$.

Let K_p be the complete graph on p vertices. It is regular of degree $p-1$. The Seidel matrix of K_p is the negation of its adjacency matrix. Therefore $SE(K_p) = 2(p-1)$ [7]. Using this in Theorem 2 we have following result.

Theorem 4: If H_a and H_b are the graphs as shown in Fig. 2, then for $p \geq 1$,
$$SE(H_a \nabla K_p) = SE(H_b \nabla K_p) = 3p + 26 + \sqrt{p^2 + 56p + 16}\ .$$

Conclusion

From Corollary 2.2, the construction of Seidel non cospectral, Seidel equienergetic graphs on equal number of vertices can be done. In particular from Theorem 3 and Theorem 4, it is easy to construct a pair of Seidel non cospectral, Seidel equienergetic n-vertex graphs for all $n \geq 12$.

Acknowledgement

The author HSR is thankful to University Grants Commission (UGC), Govt. of India for support through research grant under UPE FAR-II grant No. F 14-3/2012 (NS/PE). Another author MMG is thankful to the Karnatak University for support through UGC-UPE scholarship No. KU/Sch/UGC-UPE/2013-14/1136.

References

[1] C. Adiga, R. Balakrishnan, W. So, The skew energy of digraphs, Linear Algebra Appl. 432 (2010) 1825-1835.

[2] R. Balakrishnan, The energy of a graph, Linear Algebra Appl. 387 (2004) 287-295.

[3] M. A. Bhat, S. Pirzada, On equienergetic signed graphs, Discrete Appl. Math. 189 (2015) 1-7.

[4] A. S. Bonifacio, C. T. M. Vinagre, N. M. M. de Abreu, Constructing pairs of equienergetic and non-cospectral graphs, Appl. Math. Lett. 21 (2008) 338-341.

[5] V. Brankov, D. Stevanovic, I. Gutman, Equienergetic chemical trees, J. Serb. Chem. Soc. 69 (2004) 549-553.

[6] A. E. Brouwer, W. H. Haemers, Spectra of Graphs, Springer, Berlin, 2012.

[7] D. M. Cvetkovic, M. Doob, H. Sachs, Spectra of Graphs, Academic Press, New York, 1980.

[8] H. A. Ganie, S. Pirzada, A. Ivanyi, Energy, Laplacian energy of double graphs and new families of equienergetic graphs, Acta Univ. Sapientiae Info. 6 (2014) 89-116.

[9] K. A. Germina, K. S. Hameed, On signed paths, signed cycles and their energies, Appl. Math. Sci. 4 (2010) 3455-3466.

[10] K. A. Germina, K. S. Hameed, T. Zaslavsky, On product and line graphs of signed graphs, their eigenvalues and energy, Linear Algebra Appl. 435 (2011) 2432-2450.

[11] E. Ghorbani, On eigenvalues of Seidel matrices and Haemers conjecture, arXiv:1301.0075v1.

[12] I. Gutman, The energy of a graph, Ber. Math. Stat. Sekt. Forschungsz. Graz 103 (1978) 1-22.

[13] W. H. Haemers, Seidel switching and graph energy, MATCH Commun. Math. Comput. Chem. 68 (2012) 653-659.

[14] G. Indulal, I. Gutman, A. Vijaykumar, On the distance energy of a graph, MATCH Commun. Math. Comput. Chem. 60 (2008) 473-484.

[15] G. Indulal, A. Vijaykumar, Equienergetic self-complementary graphs, Czechoslovak Math. J. 58 (2008) 451-461.

[16] X. Li, Y. Shi, I. Gutman, Graph Energy, Springer, New York, 2012.

[17] W. Lopez, J. Rada, Equienergetic digraphs, Int. J. Pure Appl. Math. 36 (2007) 361-372.

[18] Nageswari, P. B. Sarasija, Seidel energy and its bounds, Int. J. Math. Analysis 8 (2014) 2869-2871.

[19] N. G. Nayak, Equienergetic net-regular signed graphs, Int. J. Contemp. Math. 9 (2014) 685-693.

[20] M. R. Oboudi, Energy and Seidel energy of graphs, MATCH Commun. Math. Comput. Chem. 75 (2016) 291-303.

[21] S. Pirzada, M. A. Bhat, Energy of signed digraphs, Discrete Appl. Math. 169 (2014) 195-205.

[22] H. S. Ramane, I. Gutman, M. M. Gundloor, Seidel energy of iterated line graphs of regular graphs, Kragujevac J. Math. 39 (2015) 7-12.
[23] H. S. Ramane, I. Gutman, D. S. Revankar, Distance equienergetic graphs, MATCH Commun. Math. Comput. Chem. 60 (2008) 473-484.
[24] H. S. Ramane, I. Gutman, H. B. Walikar, S. B. Halkarni, Equienergetic complement graphs, Kragujevac J. Sci. 27 (2005) 67-74.
[25] H. S. Ramane, K. C. Nandeesh, I. Gutman, X. Li, Skew equienergetic digraphs, Trans. Comb. 5 (2016) 15-23.
[26] H. S. Ramane, D. S. Revankar, I. Gutman, H. B. Walikar, Distance spectra and distance energies of iterated line graphs of regular graphs, Publ. Inst. Math. (Beograd) 85 (2009) 39-46.
[27] H. S. Ramane, H. B. Walikar, Construction of equienergetic graphs, MATCH Commun. Math. Comput. Chem. 57 (2007) 203-210.
[28] H. S. Ramane, H. B. Walikar, S. B. Rao, B. D. Acharya, P. R. Hampiholi, S. R. Jog, I. Gutman, Equienergetic graphs, Kragujevac J. Math. 26 (2004) 5-13.
[29] H. S. Ramane, H. B. Walikar, S. B. Rao, B. D. Acharya, P. R. Hampiholi, S. R. Jog, I. Gutman, Spectra and energies of iterated line graphs of regular graphs, Appl. Math. Lett. 18 (2005) 679-682.
[30] D. Stevanovic, Energy and NEPS of graphs, Lin. Multilin. Algebra 53 (2005) 67-74.
[31] L. Xu, Y. Hou, Equienergetic bipartite graphs, MATCH Commun. Math. Comput. Chem. 57 (2007) 363-370.