Bulletin of Mathematical Sciences and Applications
ISSN: 2278-9634, Vol. 15, pp 43-47
doi:10.18052/www.scipress.com/BMSA.15.43

Submitted: 2016-03-02
Revised: 2016-03-29
Accepted: 2016-03-31
Online: 2016-05-18

# Decomposition of $S^*\alpha$-continuity

V.Thiripurasundari[1,a] and , S.Suganya[2,b]

[1]PG and Research Department of Mathematics,
Sri S.Ramasamy Naidu Memorial College, Sattur-626 203, India.

[2]M.Phil Scholar, PG and Research Department of Mathematics,
Sri S.Ramasamy Naidu Memorial College, Sattur-626 203, India.

[a] thiripurasund@gmail.com, [b]suganyavsn@gmail.com

**Keywords:** semi$^*\alpha$-open, semi$^*\alpha_t$-set, semi$^*\alpha_t$-continuity.

**Abstract.**The main purpose of this paper is to introduce the concepts of $S^*\alpha_t$-sets, $S^*\alpha_t$-continuity and obtain decomposition of semi$^*\alpha$-continuity in topological spaces.

## Introduction

Levine [3], Mashhour [5] et al. and Njastad [6] introduced the topological notions of semi-open sets, pre-open sets and $\alpha$-open sets respectively. The concept of $g$-closed sets was introduced and studied by levine [4]. M.K.R.S.Veera Kumar [10] introduced the notion of $g^*$-closed sets and Govindappa Navalagi defined the concept of semi $\alpha$-open sets in topological spaces. Jafari et. al.[12] introduced and studied the notions of $\tilde{g}$-closed sets and $\tilde{g}_\alpha$-closed sets in topological spaces. Recently, Robert and Pious Missier [8] introduce a new class of sets namely semi$^*\alpha$-open sets in topological spaces.

In recent Years, the decomposition of continuity is one of the main interest for general topologists. In 1961, Levine obtained a decomposition of continuity which was later improved by Rose [9]. Tong decomposed continuity into $\alpha$-continuity and $\mathcal{A}$-continuity finally showed that his decomposition is independent of Levine's. Ganster and Reilly [2] have improved Tong's decomposition result and provided a decomposition of $\mathcal{A}$ -continuity. Ravi [11] et. al. obtained a decomposition of $\tilde{g}$-continuity. In this paper we obtain a decomposition of $S^*\alpha$-continuity in topological spaces using $S^*p$-continuity and $S^*\alpha_t$-continuity.

## Preliminaries

Throughout this paper, $(X,\tau)$ and $(Y,\sigma)$ (simply, $X$ and $Y$) denote topological space on which no separation axioms are assumed. Let $A$ be a subset of a space $X$. The closure of $A$ and the interior of $A$ are denoted by $Cl(A)$ and $Int(A)$ respectively.

**Definition 1** *A subset $A$ of a space $(X,\tau)$ is called:*

*(1) a semi-open set [3] if $A \subseteq Cl(Int(A))$ and a semi-closed set if $Int(Cl(A)) \subseteq A$,*

*(2) a pre-open set [5] if $A \subseteq Int(Cl(A))$ and a pre-closed set if $Cl(Int(A)) \subseteq A$,*

*(3) an $\alpha$-open set [6] if $A \subseteq Int(Cl(Int(A)))$ and an $\alpha$-closed set if $Cl(Int(Cl(A))) \subseteq A$,*

*(4) generalized closed [4] (briefly $g$-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open,*

*(5) a $t$-set [13] if $Int(Cl(A)) = Int(A)$,*

(6) *a $B$-set [13] if $A = M \cap N$ where $M$ is open and $N$ is a $t$-set,*

(7) *an $\alpha B$-set [1] if $A = M \cap N$ where $M$ is $\alpha$-open and $N$ is a $t$-set,*

(8) *an $\eta$-set [7] if $A = M \cap N$ where $M$ is open and $N$ is an $\alpha$-closed set,*

(9) *semi$^*\alpha$-open [8] if there is an $\alpha$-open set $U$ in $X$ such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Cl^*(\alpha Int(A))$.*

(10) *semi$^*$-preopen [8] if $A \subseteq Cl^*(pInt(A))$.*

For a subset $A$ of a space $X$, the $\alpha$-closure (resp. semi-closure, pre-closure) of $A$, denoted by $\alpha Cl(A)$ (resp.$sCl(A), pCl(A)$) is the intersection of all $\alpha$-closed (resp. semi-closed, pre-closed) subsets of $A$ containing $A$. Dually, the $\alpha$-interior (resp. semi-interior, pre-interior) of $A$, denoted by $\alpha Int(A)$ (resp. $sInt(A), pInt(A)$), is the union of all $\alpha$-open (resp. semi-open, pre-open) subsets of $X$ contained in $A$, the generalized closure of $A$ is defined as the intersection of all $g$-closed sets containing $A$ and is denoted by $Cl^*(A)$.

**Definition 2** *A function $f : X \to Y$ is said to be*

(1) *$B$-continuous [13] if $f^{-1}(V)$ is a $B$-set in $X$ for every open set $V$ of $Y$,*

(2) *$\alpha B$-continuous [1] if $f^{-1}(V)$ is an $\alpha B$-set in $X$ for every open set $V$ of $Y$,*

(3) *$\eta$-continuous [7] if $f^{-1}(V)$ is an $\eta$-set in $X$ for every open set $V$ of $Y$,*

(4) *semi$^*\alpha$-continuous [8] if $f^{-1}(V)$ is semi$^*\alpha$-open in $X$ for every open set $V$ in $Y$,*

(5) *semi$^*$-pre continuous [8] if $f^{-1}(V)$ is semi$^*$-preopen in $X$ for every open set $V$ in $Y$.*

**Remark 1** *In any space $(X, \tau)$,*

(1) *Every semi$^*\alpha$-open set is semi$^*$-preopen set but not conversely.[8]*

(2) *Every semi$^*\alpha$-continuous function is semi$^*$-pre continuous but not conversely.[8]*

**Semi$^*\alpha_t$-sets**

**Definition 3** *A subset $A$ of a topological space $(X, \tau)$ is said to be semi$^*\alpha_t$-set if $A = M \cap N$ where $M$ is a semi$^*\alpha$-open in $X$ and $N$ is a $t$-set in $X$.*
*The family of all semi$^*\alpha_t$-sets in a space $(X, \tau)$ is denoted by $S^*\alpha_t(X, \tau)$.*

**Remark 2** *The following implications are hold:*

$$\begin{tikzcd}
\eta(X) \arrow[d] & & & \\
B(X) \arrow[r] & \alpha B(X) \arrow[r] & S^*\alpha_t(X) & \\
& & S^*\alpha O(X) \arrow[u] \arrow[r] & S^*pO(X)
\end{tikzcd}$$

*where none of the these implications are reversible as shown in the following examples.*

**Example 1** *Let* $X = \{a, b, c\}$ *and* $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$. *In* $(X, \tau)$, *the set* $\{a, c\}$ *is semi*$^*\alpha_t$-*set but not semi*$^*\alpha$-*open.*

**Example 2** *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$. *In* $(X, \tau)$, *the set* $\{b, c, d\}$ *is semi*$^*\alpha_t$ *set but not* $B$-*set.*

**Example 3** *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$. *In* $(X, \tau)$, *the set* $\{b, d\}$ *is semi*$^*\alpha_t$-*set but not* $\alpha B$-*set.*

**Example 4** *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{\phi, \{a, c\}, \{a, c, d\}, X\}$. *In* $(X, \tau)$, *the set* $\{a, b, c\}$ *is semi*$^*\alpha_t$-*set but not* $\eta$-*set.*

**Remark 3** *The notions of semi*$^*\alpha_t$-*sets and semi*$^*$-*preopen sets are independent are shown in the following example.*

**Example 5** *Let* $X = \{a, b, c\}$ *and* $\tau = \{\phi, \{a, b\}, X\}$. *In* $(X, \tau)$, *the set* $\{c\}$ *is semi*$^*\alpha_t$-*set but not semi*$^*$-*preopen and also the set* $\{a, c\}$ *is semi*$^*$-*preopen but not semi*$^*\alpha_t$-*set.*

**Theorem 1** *For a subset* $A$ *of a space* $X$, *the following are equivalent:*

*(1)* $A$ *is semi*$^*\alpha$-*open*

*(2)* $A$ *is semi*$^*\alpha_t$-*set and semi*$^*$-*preopen.*

*Proof.* $(1) \Rightarrow (2)$ : The proof is obvious.
$(2) \Rightarrow (1)$ : Assume that $A$ is semi$^*\alpha_t$-set and semi$^*$-preopen in $X$. Then $A = M \cap N$ where $M$ is semi$^*\alpha$-open and $N$ is a $t$-set in $X$. By the hypothesis, $A$ is semi$^*$-preopen and we have

$$\begin{aligned}
A &\subseteq Cl^*(pInt(A)) = Cl^*(A \cap Int(Cl(A))) \\
&\subseteq Cl^*(A) \cap Cl^*(Int(Cl(A))) \subseteq Cl^*(Int(Cl(A))) \\
M \cap N \subseteq N &\subseteq Cl^*(Int(Cl(M \cap N))) \\
&= Cl^*(Int(Cl(M)) \cap Int(Cl(N))) \\
&\subseteq Cl^*(Int(Cl(M))) \cap Cl^*(Int(Cl(N))) \\
&\subseteq Cl^*(Int(Cl(N))) = Cl^*(Int(N)) \subseteq Cl^*(\alpha Int(N))
\end{aligned}$$

Since $M$ is semi$^*\alpha$-open in $X$, we have $M \subseteq Cl^*(\alpha Int(M))$

$$M \cap N \subseteq Cl^*(\alpha Int(M \cap N))$$

Therefore $A$ is semi$^*\alpha$-open in $X$.

**semi**$^*\alpha_t$**-continuity**

**Definition 4** *A function* $f : X \to Y$ *is said to be semi*$^*\alpha_t$-*continuous if* $f^{-1}(V)$ *is semi*$^*\alpha_t$-*set in* $X$ *for every open set* $V$ *of* $Y$.

**Remark 4** *The following implications are hold:*

$$\begin{tikzcd}
\eta - continuity \arrow[d] & & \\
B - continuity \arrow[r] & \alpha B - continuity \arrow[r] & S^*\alpha_t - continuity \\
& S^*p - continuity & S^*\alpha - continuity \arrow[l] \arrow[u]
\end{tikzcd}$$

*where none of these implications are reversible as shown in the following examples.*

**Example 6** *Let* $X = \{a, b, c\}$ *and* $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$, *and* $\sigma = \{\phi, \{q\}, \{r\}, \{q, r\}, X\}$. *Define* $f : (X, \tau) \to (Y, \sigma)$ *as follows:* $f(a) = p, f(b) = q, f(c) = r$. *Then* $f$ *is semi*$^*\alpha_t$*-continuous but not semi*$^*\alpha$*-continuous.*

**Example 7** *Let* $X = \{a, b, c\}$ *and* $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, X\}$ *and* $\sigma = \{\phi, \{p\}, \{r\}, \{p, r\}, Y\}$. *Define* $f : (X, \tau) \to (Y, \sigma)$ *as follows:* $f(a) = p, f(b) = r, f(c) = q$. *Then* $f$ *is semi*$^*\alpha_t$*-continuous but not* $B$*-continuous.*

**Example 8** *Let* $X = \{a, b, c, d\}$ *and* $Y = \{p, q, r, s\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ *and* $\sigma = \{\phi, \{s\}, \{p, s\}, \{q, s\}, \{p, q, s\}, Y\}$. *Define* $f : (X, \tau) \to (Y, \sigma)$ *as follows:* $f(a) = r, f(b) = s, f(c) = q, f(d) = p$. *Then* $f$ *is* $s^*\alpha_t$*-continuous but not* $\alpha B$*-continuous.*

**Example 9** *Let* $X = \{a, b, c\}$ *and* $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ *and* $\sigma = \{\phi, \{p\}, \{p, q\}, \{p, r\}, Y\}$. *Define* $f : (X, \tau) \to (Y, \sigma)$ *as follows:* $f(a) = q, f(b) = r, f(c) = p$. *Then* $f$ *is semi*$^*\alpha_t$*-continuous but not* $\eta$*-continuous.*

**Remark 5** *The following examples show that the concept of semi*$^*\alpha_t$*-continuity and semi*$^*$*-pre continuity are independent.*

**Example 10** *Let* $X = \{a, b, c\}$ *and* $Y = \{p, q, r\}$, $\tau = \{\phi, \{a, b\}, X\}$ *and* $\sigma = \{\phi, \{q\}, \{r\}, \{p, q\}, \{q, r\}, Y\}$. *Define* $f : (X, \tau) \to (Y, \sigma)$ *as follows* $f(a) = r, f(b) = q, f(c) = p$. *Then* $f$ *is semi*$^*$*-pre continuous but not semi*$^*\alpha_t$*-continuous.*

**Example 11** *Let* $X = \{a, b, c\}$ *and* $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ *and* $\sigma = \{\phi, \{p\}, \{p, q\}, Y\}$. *Define* $f : (X, \tau) \to (Y, \sigma)$ *as follows* $f(a) = r, f(b) = q, f(c) = p$. *Then* $f$ *is semi*$^*\alpha_t$*-continuous but not semi*$^*$*-pre continuous.*

**Theorem 2** *For a function* $f : X \to Y$, *the following are equivalent:*

*(1)* $f$ *is semi*$^*\alpha$*-continuous.*

*(2)* $f$ *is semi*$^*\alpha_t$*-continuous and semi*$^*$*-pre continuous.*

*Proof.* The proof follows from Theorem 1.

**References**


[1] AL-Nashef B, *A decomposition of $\alpha$-continuity and semi-continuity*, Acta Math. Hungar., 97(1-2)(2002), 115-120.

[2] Ganster M and Reilly I. L, *A decomposition of continuity,* Acta Math. Hungar, 56(1990), 299-301.

[3] Levine N, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70 (1963), 36-41.

[4] Levine N, *Generalized Closed Sets in Topology*, Rend. Circ. Mat. Palermo, 19(2) (1970), 89-96.

[5] Mashhour A. S, El-Monsef M. A and El-Deep S. N, *On pre-continuous and weak pre-continuous mappings*, Proc. Math. and Phys. Soc. Egypt, 53(1982), 47-53.

[6] Njastad O, *On some classes of nearly open sets*, Pacific J. Math, 15(1965), 961-970.

[7] Noiri T, Sayed O.R, *On decomposition of continuity*, Acta Math. Hungar, 111(1-2)(2006), 1-8.

[8] Robert A, and Pious Missier S, *Semi-Star-Alpha-Open Sets and Associated Functions,* International Journal of Computer Applications, 104(16)(2014), 24-29.

[9] Rose D, *On Levine's decomposition of continuity*, Canad. Math. Bull, 21(1978), 477-481.

[10] Rose Mary S, Arivu Chelvam A, and Ravi O, *Contra $g^*$-continuity and Separation Axioms*, International Journal of Current Research in Science and Technology, 1(4)(2015), 25-31.

[11] Rose Mary S, Kavitha T, and Ravi O, *Decompositions of $g$-continuity*, International Journal of Mathematics and its applications, 3(2)(2015), 1-8.

[12] Thivagar M.L and Nirmala Rebecca P, *On Topological closed graphs in the new form,* Jordan Journal of Mathematics and Statistics 5(4), 2012, 251 - 266.

[13] Tong J, *On decomposition of continuity in topological spaces,* Acta Math. Hungar, 54 (1989), 51-55.