

Decomposition of $S^* \alpha$ -continuity

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Abstract. The main purpose of this paper is to introduce the concepts of $S^* \alpha_t$ -sets, $S^* \alpha_t$ -continuity and obtain decomposition of semi $^* \alpha$ -continuity in topological spaces.

Introduction

Levine [3], Mashhour [5] et al. and Njastad [6] introduced the topological notions of semi-open sets, pre-open sets and α -open sets respectively. The concept of g -closed sets was introduced and studied by Levine [4]. M.K.R.S.Veera Kumar [10] introduced the notion of g^* -closed sets and Govindappa Navalagi defined the concept of semi α -open sets in topological spaces. Jafari et. al.[12] introduced and studied the notions of \tilde{g} -closed sets and \tilde{g}_α -closed sets in topological spaces. Recently, Robert and Pious Missier [8] introduce a new class of sets namely semi $^* \alpha$ -open sets in topological spaces.

In recent Years, the decomposition of continuity is one of the main interest for general topologists. In 1961, Levine obtained a decomposition of continuity which was later improved by Rose [9]. Tong decomposed continuity into α -continuity and \mathcal{A} -continuity finally showed that his decomposition is independent of Levine's. Ganster and Reilly [2] have improved Tong's decomposition result and provided a decomposition of \mathcal{A} -continuity. Ravi [11] et. al. obtained a decomposition of \tilde{g} -continuity. In this paper we obtain a decomposition of $S^* \alpha$ -continuity in topological spaces using $S^* p$ -continuity and $S^* \alpha_t$ -continuity.

Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (simply, X and Y) denote topological space on which no separation axioms are assumed. Let A be a subset of a space X . The closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively.

Definition 1 A subset A of a space (X, τ) is called:

- (1) a semi-open set [3] if $A \subseteq Cl(Int(A))$ and a semi-closed set if $Int(Cl(A)) \subseteq A$,
- (2) a pre-open set [5] if $A \subseteq Int(Cl(A))$ and a pre-closed set if $Cl(Int(A)) \subseteq A$,
- (3) an α -open set [6] if $A \subseteq Int(Cl(Int(A)))$ and an α -closed set if $Cl(Int(Cl(A))) \subseteq A$,
- (4) generalized closed [4] (briefly g -closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,
- (5) a t -set [13] if $Int(Cl(A)) = Int(A)$,

- (6) a B -set [13] if $A = M \cap N$ where M is open and N is a t -set,
- (7) an αB -set [1] if $A = M \cap N$ where M is α -open and N is a t -set,
- (8) an η -set [7] if $A = M \cap N$ where M is open and N is an α -closed set,
- (9) $\text{semi}^* \alpha$ -open [8] if there is an α -open set U in X such that $U \subseteq A \subseteq Cl^*(U)$ or equivalently if $A \subseteq Cl^*(\alpha Int(A))$.
- (10) semi^* -preopen [8] if $A \subseteq Cl^*(pInt(A))$.

For a subset A of a space X , the α -closure (resp. semi-closure, pre-closure) of A , denoted by $\alpha Cl(A)$ (resp. $sCl(A)$, $pCl(A)$) is the intersection of all α -closed (resp. semi-closed, pre-closed) subsets of A containing A . Dually, the α -interior (resp. semi-interior, pre-interior) of A , denoted by $\alpha Int(A)$ (resp. $sInt(A)$, $pInt(A)$), is the union of all α -open (resp. semi-open, pre-open) subsets of X contained in A , the generalized closure of A is defined as the intersection of all g -closed sets containing A and is denoted by $Cl^*(A)$.

Definition 2 A function $f : X \rightarrow Y$ is said to be

- (1) B -continuous [13] if $f^{-1}(V)$ is a B -set in X for every open set V of Y ,
- (2) αB -continuous [1] if $f^{-1}(V)$ is an αB -set in X for every open set V of Y ,
- (3) η -continuous [7] if $f^{-1}(V)$ is an η -set in X for every open set V of Y ,
- (4) $\text{semi}^* \alpha$ -continuous [8] if $f^{-1}(V)$ is $\text{semi}^* \alpha$ -open in X for every open set V in Y ,
- (5) semi^* -pre continuous [8] if $f^{-1}(V)$ is semi^* -preopen in X for every open set V in Y .

Remark 1 In any space (X, τ) ,

- (1) Every $\text{semi}^* \alpha$ -open set is semi^* -preopen set but not conversely.[8]
- (2) Every $\text{semi}^* \alpha$ -continuous function is semi^* -pre continuous but not conversely.[8]

$\text{Semi}^* \alpha_t$ -sets

Definition 3 A subset A of a topological space (X, τ) is said to be $\text{semi}^* \alpha_t$ -set if $A = M \cap N$ where M is a $\text{semi}^* \alpha$ -open in X and N is a t -set in X .

The family of all $\text{semi}^* \alpha_t$ -sets in a space (X, τ) is denoted by $S^* \alpha_t(X, \tau)$.

Remark 2 The following implications are hold:

$$\begin{array}{ccccccc}
 & & \eta(X) & & & & \\
 & & \downarrow & & & & \\
 B(X) & \longrightarrow & \alpha B(X) & \longrightarrow & S^* \alpha_t(X) & & \\
 & & & & \uparrow & & \\
 & & & & S^* \alpha O(X) & \longrightarrow & S^* pO(X)
 \end{array}$$

where none of these implications are reversible as shown in the following examples.

Example 1 Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$. In (X, τ) , the set $\{a, c\}$ is $\text{semi}^*\alpha_t$ -set but not $\text{semi}^*\alpha$ -open.

Example 2 Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$. In (X, τ) , the set $\{b, c, d\}$ is $\text{semi}^*\alpha_t$ set but not B -set.

Example 3 Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$. In (X, τ) , the set $\{b, d\}$ is $\text{semi}^*\alpha_t$ -set but not αB -set.

Example 4 Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{a, c\}, \{a, c, d\}, X\}$. In (X, τ) , the set $\{a, b, c\}$ is $\text{semi}^*\alpha_t$ -set but not η -set.

Remark 3 The notions of $\text{semi}^*\alpha_t$ -sets and semi^* -preopen sets are independent as shown in the following example.

Example 5 Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a, b\}, X\}$. In (X, τ) , the set $\{c\}$ is $\text{semi}^*\alpha_t$ -set but not semi^* -preopen and also the set $\{a, c\}$ is semi^* -preopen but not $\text{semi}^*\alpha_t$ -set.

Theorem 1 For a subset A of a space X , the following are equivalent:

- (1) A is $\text{semi}^*\alpha$ -open
- (2) A is $\text{semi}^*\alpha_t$ -set and semi^* -preopen.

Proof. (1) \Rightarrow (2) : The proof is obvious.

(2) \Rightarrow (1) : Assume that A is $\text{semi}^*\alpha_t$ -set and semi^* -preopen in X . Then $A = M \cap N$ where M is $\text{semi}^*\alpha$ -open and N is a t -set in X . By the hypothesis, A is semi^* -preopen and we have

$$\begin{aligned} A &\subseteq Cl^*(pInt(A)) = Cl^*(A \cap Int(Cl(A))) \\ &\subseteq Cl^*(A) \cap Cl^*(Int(Cl(A))) \subseteq Cl^*(Int(Cl(A))) \\ M \cap N \subseteq N &\subseteq Cl^*(Int(Cl(M \cap N))) \\ &= Cl^*(Int(Cl(M)) \cap Int(Cl(N))) \\ &\subseteq Cl^*(Int(Cl(M))) \cap Cl^*(Int(Cl(N))) \\ &\subseteq Cl^*(Int(Cl(N))) = Cl^*(Int(N)) \subseteq Cl^*(\alpha Int(N)) \end{aligned}$$

Since M is $\text{semi}^*\alpha$ -open in X , we have $M \subseteq Cl^*(\alpha Int(M))$

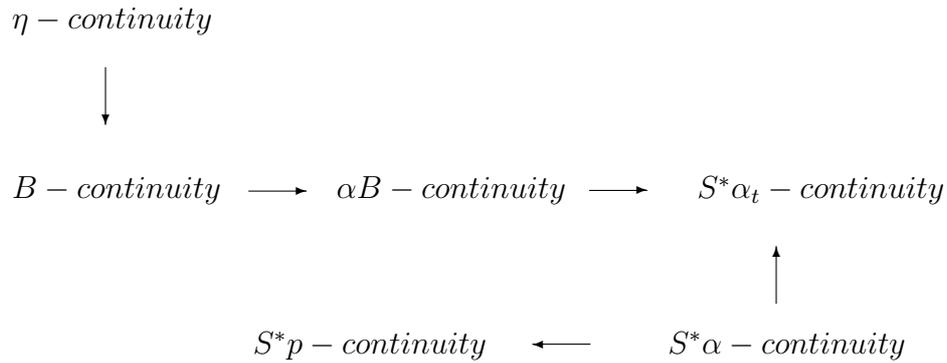
$$M \cap N \subseteq Cl^*(\alpha Int(M \cap N))$$

Therefore A is $\text{semi}^*\alpha$ -open in X .

semi^{*} α_t -continuity

Definition 4 A function $f : X \rightarrow Y$ is said to be $\text{semi}^*\alpha_t$ -continuous if $f^{-1}(V)$ is $\text{semi}^*\alpha_t$ -set in X for every open set V of Y .

Remark 4 The following implications are hold:



where none of these implications are reversible as shown in the following examples.

Example 6 Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$, and $\sigma = \{\phi, \{q\}, \{r\}, \{q, r\}, X\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ as follows: $f(a) = p, f(b) = q, f(c) = r$. Then f is $\text{semi}^* \alpha_t$ -continuous but not $\text{semi}^* \alpha$ -continuous.

Example 7 Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{p\}, \{r\}, \{p, r\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ as follows: $f(a) = p, f(b) = r, f(c) = q$. Then f is $\text{semi}^* \alpha_t$ -continuous but not B -continuous.

Example 8 Let $X = \{a, b, c, d\}$ and $Y = \{p, q, r, s\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{s\}, \{p, s\}, \{q, s\}, \{p, q, s\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ as follows: $f(a) = r, f(b) = s, f(c) = q, f(d) = p$. Then f is $s^* \alpha_t$ -continuous but not αB -continuous.

Example 9 Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{p\}, \{p, q\}, \{p, r\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ as follows: $f(a) = q, f(b) = r, f(c) = p$. Then f is $\text{semi}^* \alpha_t$ -continuous but not η -continuous.

Remark 5 The following examples show that the concept of $\text{semi}^* \alpha_t$ -continuity and semi^* -pre continuity are independent.

Example 10 Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{q\}, \{r\}, \{p, q\}, \{q, r\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ as follows $f(a) = r, f(b) = q, f(c) = p$. Then f is semi^* -pre continuous but not $\text{semi}^* \alpha_t$ -continuous.

Example 11 Let $X = \{a, b, c\}$ and $Y = \{p, q, r\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{p\}, \{p, q\}, Y\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ as follows $f(a) = r, f(b) = q, f(c) = p$. Then f is $\text{semi}^* \alpha_t$ -continuous but not semi^* -pre continuous.

Theorem 2 For a function $f : X \rightarrow Y$, the following are equivalent:

- (1) f is $\text{semi}^* \alpha$ -continuous.
- (2) f is $\text{semi}^* \alpha_t$ -continuous and semi^* -pre continuous.

Proof. The proof follows from Theorem 1.

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