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FIXED POINT THEOREMS FOR COMPATIBLE MAPPINGS OF TYPE (R) IN MENGER SPACES

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ABSTRACT The aim of this paper is to introduce the concept of weak compatible mappings of type (R) in menger spaces and to prove a fixed point theorem for compatible mappings of type (R) in menger spaces.

1. Introduction

Rohen Singh, M.R & Shambhu introduced the concept of compatible mappings of type (C) by combinging the definition of compatible mappings and compatible mapping of type (P) and later on it is renamed as compatible mappings of type (R) [14]. Later it is extensively developed by Rohen and many others. In the last decade a number of authors [16], [17], [18] have studied the aspects of compatible mappings of type (R).

The concept of probabilistic metric space were first introduced and studied by Menger [11] which is a generalization of metric spaces. These spaces and their applications were study by Bharuch-Read [1] Chang [4] Ciric [6] Chamola [3] Cain & Kasriel [2] Debeic and Sarapa [7] Egbert [8] Hadzic [9] and Hicks [10]

Recently Hadzic [9] Radu [13] Cho et al [5] Pathak et al [12] Singh and Pant [19] Spacek [20] Vasuki [21] and many others proved Common fixed point theorems in menger spaces.

Definition 2.1 : A probabilistic metric space is a pair(X,F) where X is a non-empty set and F is a mapping fromXxX to L.

For (u,v)∈XxX The distribution function F(u,v) is denoted by Fu,v the functions Fu, v are assumed to satisfy the following conditions.

(P1) Fu,v(x)=1 for every x,o iff u=v

(P2) Fu,v(o)=o for every u,v ∈ X

(P3) Fu,v(x) Fv,u(x)=Fv,u for every u,v ∈ X

(P4) if Fu,v (x)=1 and F v,w(y)=1

Then Fu,w (x+y)=1 for every u,v W ∈ X

Definition 2.2 A manger. Space is a tripat(X,F,t) where(X,F) is a PM – Space and t is T- norm with the following condition.

(P5)Fu,w (x+y)≥t[Fuv(x),Fv,w(y)]

For every u,v,w ∈ X and x,y ∈ R+

Definition 2.3 : Let (X,F,t) be a Merger space with the continuous T- norm t,

(i) A sequence $\{p_n\}$ in X is said to be convergent to a point P ∈ X if for every ε>0 and λ>0 there exists an integer N=N(ε,λ) such that p_n=Up(ε,λ) for Fp1 p_n(ε)>L-λ, for all n≥N, we rerate Pn→P as n→∞ or $\lim_{n\to\infty} p_n=p$

(ii) A sequence {pn} of points in X is said to be a Cauchy sequence if for every ε>0 and λ>0 there exists an integer N=N(ε,λ) ≥0 such that Fpnpm(ε)>1-λ for all m,n≥N

(iii) The Merger space (X,F,t) is said to be complete if every Cauchy sequence in X converges to a point in X.

Definition 2.4: Let (X,F,t) be a Merger space such that the T-norm is continuous and A,S be mappings from X into itself. A and S are said to be compatible if

$\lim_{n\to\infty} FSAx_n\, SAx_{n\,(x)} =1$

For all x>0 wherever {Xn} is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$

For some z∈X.
We introduce the following definitions
Definition 2.5 : Let(X,F,t) be a Merger space such that T-norm t is continuous and A, S be mapping from X into itself
A and S are said to be compatible of type (R) if
$\lim_{n\to\infty} F_{AS\,xn,SAxn(x)}=1$ and
$\lim_{n\to\infty} F_{AA\,xn,SS\,xn\,(x)}=1$
For all x>0 whenever $\{x_n\}$is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$
For some z∈X.
Definition 2.6: Let(X,F,t) be a Merger space such that the T- norm is continuous and A,S be mappings from X into itself.
A and S are said to be weak compatible mappings of type (R) if.
$\lim_{n\to\infty} F_{AA\,xn,SS\,xn\,(x)} \geq \lim_{n\to\infty} F_{SAxn,ASxn(x)}$ and $\lim_{n\to\infty} F_{SAxn,ASxn(x)} \geq \lim_{n\to\infty} F_{SSxn\,AAxn(x)}$
For all x>0 whenever {xn} is a sequence in X such that $\lim_{n\to\infty} A\,x_n = \lim_{n\to\infty} Sx_n = z$ for some z∈X.
The following proposition 2.7 and 2.8 show that definition 2.4 and 2.5 are equivalent under some conditions.
Proposition 2.7 :Let(X,F,t) be Merger space such that the T-norm is continuous and t(x,x) ≥x for all x ∈ [0, 1] and let A,S:→Xbe continuous mappings. Then A and S are compatible of type (R).
Proposition 2.8:Let (X,F,t) be merger space such that the T- norm t is continuous t(x,x) ≥x and for all x∈ [0, 1] and let A,S:X→X be compatible mappings of type (R) .
If one of A and S is continuous, then A and S are compatible.
From proposition 2.7 and 2.8 we have
Proposition 2.9 Let (X,F,t) be merger space such that the T –norm t is continuous and t(x,x) ≥x for all x∈ [0, 1] and let A,S: X→X be compatible mappings. Then A and S are compatible if and only if they are compatible of type (R).
The following proposition, show that definition 2.4,2.5 and 2.6 are equivalent under. Some condition but first we have.
Proposition 2.10: Let (X,F,t) be a Merger space such that the T- norm t is continuous and t(x,x) ≥x for all x∈ [0, 1] and let A,S: X→X be compatible mappings. Then A and S are weak compatible mappings of type (R) if they are compatible mappings of type (R).
Proposition 2.11 : Let (X,F,t) be a Merger Space such that the T-norm t is continuous and t (x,x) ≥ x for all x∈ [0, 1] and A, S : X → X be continuous mapping s. If A and S are weak compatible mappings of type (R), then they are compatible mappings of type (R).
Proposition 2.12: Let (X, F, t) be Merger Space, such that the T-norm t is continuous and t(x,x) ≥ x for all x∈ [0, 1] and A,S : X→ X be weak compatible of type (R). If one of A and S is continuous, then A and S are compatible mappings.
As a direct consequence of proposition 2.7, 2.10 and 2.12 we have the following
Proposition 2.13 : Let (X, F, t) be a Merger Space such that T-norm t is continuous and t (x, x) ≥ x for all x ε [0, 1] and A, S : X → X be continuous mappings. Then A and S are compatible if and only if they are weak compatible mappings of type (R).
By using proposition 2.10, 2.11 and 2.13 we have the following.
Proposition 2.14: Let (X, F, t) be a Merger Space such that the T-norm t is continuous and t (x, x) ≥ x for all x ε [0,1] and A, S : X → X be compatible mappings then of type (R) if and only if they are weak compatible mappings of type (R)

i) A and S are compatible mappings of type (R) if and only if they are weak compatible mappings of type (R)
ii) A and S are compatible mappings if and only if they are weak compatible mappings of type (R).

Next we give two propositions of weak compatible mappings of type (R) on a Merger Space for our main theorem.

Proposition 2.15: Let (X, F, t) be a Merger Space such that the T-norm t is continuous and t (x, x) ≥ x for all x ∈ [0, 1] and A, S : X → X be mappings of type (R) and Az = Sz for some Z ∈ X then AAz = ASz = SA =SSz.

Proposition 2.16: Let (X, F, t) be Merger Space such that the t-norm t is continuous and t(x, x) ≥ x for all x ε [0, 1] and A.S : X → X be mappings. Let A and S be weak compatible mappings of type (R) and $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some z∈X

Then we have

(i) $\lim_{n\to\infty} SAx_n$ = Az if A is continuous

(ii) $\lim_{m\to\infty} ASx_n$ = Sz if S is continuous

(iii) ASz = SAz and Az = Az = Sz if A and S are continuous.Lemma 2.17 : Let $\{x_n\}$ be a sequence in a Merger Space (X, F, t) where t is a continuous T-norm and t (x, x) ≥ x for all x ∈ [0, 1]. If there exist a constant k ∈ (0, 1) Such that $Fx_n, x_{n+1}(kx) \geq Fx_{n-1}, x_n(x)$ for all x > 0 and n ∈ N, then $\{x_n\}$ is a Cauchy sequence in X.

2 Ours main Theorem

Theorem 3.1.: Let (X,F,t) be a complete Merger Space with t(x,y) min (x,y) for all x, y ϵ [0, 1] and P,Q,R,A,B,S,T be mappings from X itself such that

(3.1) P(X) ∪QAB(X) ⊂RST (X),

(3.2) The pairs P, QAB and P, RST are weak compatible of type (R),

(3.3) Q,A,B are continuous,

(3.4) Fz, QABz(x) ≤ Fz, RSTz(x)

(3.5) $[F_{pu}, Pv\ (kx)]^2 \geq \min \{[F_{QABu}, RSTv\ (x)]^2, [F_{QAPu(x)}, F_{RSTv(x)Pv}(x)$

$\geq \min\{[F_{QABu}, RSTv\ (x)]^2, [F_{QAPu(x)}, F_{RSTv(x)Pv}(x)$

$F_{QABu}, RSTv\ (x), F_{QABu,Pu(x)}, F_{QABu}, F_{RSTv,}Pv(x),$

$F_{QABu}, RSTv\ (2x), F_{QABu,Pv(x)}, F_{QABu}, RSTv\ (x)\ F_{RSTv,}Pu(x),$

$F_{QABu}, Pv\ (2x), F_{RSTv,Pv(x)}, F_{QABu}, Pu\ (x)\ F_{RSTv,}Pu(x),$

$F_{QABu}, Pv\ (2x), F_{RSTv,}Pv(x)\}$

For all u, v, ϵ, X and x ≥ o, where k ϵ (0, 1).

Then P, Q, R, A, B, S and T have a unique common fixed point.

Proof : For any xo ϵ X, there exist a point x1, we can choose a point x2 in X such that Px0 = RSTx1. For this point x1, we can choose a point x2 ϵ x, such that Px1 = QABx2 and so on, in this manner we can define a sequence {yn} in X such that

$y_{2n} + RSTx_{2n+1} = Px_{2n}$

$y_{2n+1} = QABx_{2n+2} = Px_{2n+1}$ n=0,1,2........

Now we shall prove $Fy_{2n}, Y_{2n+1}(kx) \geq Fy_{2n+1}, Y2n(x)$, for all x > 0, where k ϵ (0,1). Suppose that $Fy_{2n}, y_{2n+1}(kx) < Fy_{2n-1}, y_{2n}(x)$. Then by using condition (3.5) and $Fy_{2n}, Y_{2n+1}(kx) \leq Fy2n, y_{2n+1}(x)$, we have

$[Fy_{2n}, y_{2n+1}(kx)]^2 = [FPx_{2n}, Px_{2n}+1(kx)]^2 \geq \min [Fy_{2n-1}, y_{2n}(x)]^2, Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x),$

$Fy_{2n-1}, y_{2n}(x), Fy_{2n-1}, y_{2n}(x), Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x),$

$Fy_{2n-1}, y_{2n}(x), Fy_{2n-1}, y_{2n-1}(2x), Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n}(x),$

$Fy_{2n-1}, y_{2n+1}(2x), Fy_{2n}, y_{2n}(x), Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n}(x),$

$Fy_{2n-1}, y_{2n+1}(2x), Fy_{2n}, y_{2n-1}(x)\}$

$\geq \min \{[Fy_{2n-1}, y_{2n}(x), Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x), [Fy_{2n-1}, y_{2n}(x)]^2, Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x),$

$Fy_{2n-1}, y_{2n}(x), t(Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x), Fy_{2n-1}, y_{2n}(x), t(Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x),$

$Fy_{2n-1}, y_{2n}(x), t(Fy_{2n-1}, y_{2n}(x), Fy_{2n}, y_{2n+1}(x), Fy_{2n}, y_{2n+1}(x)\}$

$\geq \min \{[Fy_{2n}, y_{2n+1}(kx)]^2, [Fy_{2n}, y_{2n+1}(kx)]^2,$

$[Fy_{2n}, y_{2n+1}(kx)]^2, [Fy_{2n}, y_{2n+1}(kx)]^2, [Fy_{2n}, y_{2n+1}(kx)]^2,$

$Fy_{2n}, y_{2n+1}(kx), Fy_{2n}, y_{2n+1}(kx), Fy_{2n}, y_{2n+1}(kx), [Fy_{2n}, y_{2n+1}(kx)]^2 = [Fy_{2n}, y_{2n+1}(kx)]^2$

Which is a contradiction. Thus we have $Fy_{2n}, y_{2n+1}(kx) \geq Fy_{2n-1}, y_{2n}(x)$. Similarly we have also $Fy_{2n-1}, y_{2n+1}(kx) \geq Fy_{2n},y_{2n+1}(kx)$. Therefore, by lemma (2.17), $\{y_n\}$ is a Cauchy sequence in X. Since the Menger space (X, F, t) is complete, $\{y_n\}$ converges to a point z in X, and the subsequences $\{Px_{2n}\}$, $\{QABx_{2n+1}\}$, $\{RSTx_{2n+1}\}$ and $\{Px_{2n}\}$ of $\{y_{2n}\}$ also converges to z.

Now suppose that Q, A, B are continuous, since P and QAB are weak compatible of type (R), it follows from proposition.

$PQABx_{2n+1}$ and $(QAB)^2x_{2n+1} \to (QAB)z$, as $n \to \infty$.

Now putting $u = (QAB)x_{2n+1}$ and $v = x_{2n+1}$ in condition (3.5), we have

$$[F_{P(QAB)x_{2n+1}}, Px_{2n+1}(kx)]^2 \geq \min [F_{(QAB)2_{x_{2n+1}}}, (RST)\, x_{2n+1}(x)]^2$$
$$F_{(QAB)2_{x_{2n+1}}}, P(QAB)x_{2n+1}(x), F_{(RST)x_{2n+1}}, Px_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, (RST)x_{2n+1}(x), F_{(QAB)x_{2n+1}}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, (RST)x_{2n+1}(x), F_{(RST)x_{2n+1}}, Px_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, (RST)x_{2n+1}(x), F_{(QAB)2_{x_{2n+1}}}, Px_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, (RST)x_{2n+1}(x), F_{(RST)x_{2n+1}}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, Px_{2n+1}(2x), F(RST)x_{2n+1}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, P(QAB)x_{2n+1}(x), F_{(RST)x_{2n+1}}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, Px_{2n+1}(2x), F_{(RST)x_{2n+1}}, Px_{2n+1}(x),$$

Letting $n\to\infty$ we have$=[F_{(QAB)z,z(x)}]$

Which is a contradiction. Therefore we have (QAB) z = z. Since $P(x) \subset RST(x)$,

By using condition (3.4) we have $F_{z,RSTz(x)} \leq F_{z,QABz(x)}$

Which implies that RSTz = z because QABz = z.

Again put $u = QABx_{2n+1}$ and v = z in condition (3.5), we have

$$[F_{P(QAB)x_{2n+1}}, P_z(kx)]^2 \geq \min [F_{(QAB)2_{x_{2n+1}}}, RSTz_{(x)}]^2$$
$$F_{(QAB)2_{x_{2n+1}}}, P(QAB)x_{2n+1}(x), F_{RSTz}, P_z(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, RSTz\ (x), F_{(QAB)2_{x_{2n+1}}}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, RSTz\ (x), F_{RSTz}, P_z(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, RSTz\ (x), F_{(QAB)2_{x_{2n+1}}}, P_z(2x),$$
$$F_{(QAB)2_{x_{2n+1}}}, RSTz\ (x), F_{RSTz}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, Pz\ (2x), F_{RSTz}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, P(QAB)x_{2n+1}(x), F_{RSTz}, P(QAB)x_{2n+1}(x),$$
$$F_{(QAB)2_{x_{2n+1}}}, Pz\ (2x), F_{RSTz}, Pz\ (x),$$

By taking limit $n\to\infty$, we have$=\{F_{z,Pz(x)}\}2$

Which is a contradiction, therefore Pz = z. Now we show that Az = z infact by condition (3.5), and by putting u = Az and v = z, we have

$$[F_{P(A)z,Pz(kx)}]^2 \geq \min [\{F_{(QAB)Az,RSTz(x)}\}^2,$$
$$F_{(QAB)Az,P(A)z(x)}.\ F_{RSTz,P(A)z(x)},$$
$$F_{(QAB)Az,RSTz(x)}.\ F_{(QAB)Az,P(A)z(x)},$$
$$F_{(QAB)Az,RSTz(x)}.\ F_{RSTz,Pz(x)},$$
$$F_{(QAB)Az,RSTz(x)}.\ F_{(QAB)Az,Pz(2x)},$$
$$F_{(QAB)Az,RSTz(x)}.\ F_{RSTz,P(A)z(x)},$$
$$F_{(QAB)Az,Pz(2x)}.\ F_{RSTz,P(A)z(x)},$$
$$F_{(QAB)Az,P(A)z(x)}.\ F_{RSTz,P(A)z(x)},$$
$$F_{(QAB)Az,Pz(2x)}.\ F_{RSTz,Pz(x)}],$$
$$= \{F_{Az,z(x)}\}^2$$

Which is a contradiction, therefore Az = z. Similarly, if e put u = Bz and v = z, we obtain Bz = z and, if we put u = Qz and v = z we have Qz =z.

Similarly, we can show that Rs = z, Sz = z and Tz =z.

Therefore combining above results, we have Pz = Az = Bz = Qz = Rz =Sz =Tz =z, Hence z is a common fixed point of P,A,B,Q,R,S and T.

It follows easily from (3.5) that z is a unique common fixed point of P,A,B,Q,R,S and T.

Theorem 3.2 : Let P,A,B,Q,R,S and T be mappings from a complete metric space (x,d) into itself such that

(3.2.1) P(X) ⊂RST (X) and P(X)⊂ QAB (X),

(3.2.2) Q,A,B are continuous,

(3.2.3) the pairs {P,QAB} and {P,RST} are weakly compatible of type (*R*)

(3.2.4) PA = AP, PB = BP, PS = SP, ……… PQ = QP,

(3.2.5) d{x,QABx(x)} ≥d{x,RSTx(x)},

(3.2.6) d2(Px,Py)≤ k. max {d2(QABx, RSTy). d(QABx,Px). d(RSTy, Py)
d(QABx,RSTy).d (QABx,Px),d(QABx,RSTy). d (RSTy, Py),
½ d(QABx,RSTy).d (QABx,Py),d(QABx,RSTy). d (RSTy, Px),
½ d(QABx,Py).d (RSTy,Px),d(QABx,RSTx). d (RSTy, Px),
½ d(QABx,Py).d (RSTy,Py)

For all x,y in X, where k∈ (0,1). Then P, Q, A, B, R, S And T have A unique common fixed point in x.

Theorem 3.2 can be proved in similar manner as Theorem 3.1.

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