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The Amazing Proof of The Erdős-Straus Conjecture

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Abstract: In this paper we present the proof of The Erdős-Straus Conjecture.

1.INTRODUCTION

The search for expansions of rational numbers as sums of unit fractions dates to the mathematics of ancient Egypt, in which Egyptian fraction expansions of this type were used as a notation for recording fractional quantities. The Egyptians produced tables such as the Rhind Mathematical Papyrus 2/n table of expansions of fractions of the form 2/n, most of which use either two or three terms. Egyptian fractions typically have an additional constraint, that all of the unit fractions be distinct from each other, but for the purposes of the Erdős–Straus conjecture this makes no difference: if 4/n can be expressed as a sum of at most three unit fractions, it can also be expressed as a sum of at most three distinct unit fractions. [1]

The Erdős–Straus Conjecture concerns the Diophantine Equations.

One important topic in number theory is the study of Diophantine equations, equations in which only integer solutions are permitted. The type of Diophantine equation discussed in this paper concerns Egyptian fractions, which deal with the representation of rational numbers as the sum of three unit fractions. [2]

2.THE PROOF OF THE ERDŐS-STRAUS CONJECTURE

Conjecture (Erdős– Straus Conjecture) For all $n \in \{2,3,4,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$: $\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

For all n $\in \{2,3,4,\dots\}$ and some a,b,c $\in \{1,2,3,\dots\}$:

$$\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \Rightarrow 4abc=n(ab+ac+bc). \tag{1}$$

For n=2 ans for a=1 and for b=c=2.

$\frac{4}{n}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

For all n $\in \{4,6,8,\dots\}$ and some $a,b,c \in \{1,2,3,\dots\}$

$$\left[\frac{4}{n}=\frac{1}{\frac{n}{2}}+\frac{1}{\frac{n+2}{2}}+\frac{1}{\frac{n(n+2)}{4}} \wedge \frac{n}{2}=a \wedge \frac{n+2}{2}=b \wedge \frac{n(n+2)}{4}=c\right]. \spadesuit$$

From (1) follows that for all n $\in \{3,5,7,\dots\}$ and for some $m,\ a,b,c \in \{1,2,3,\dots\}$

$$n=\frac{(4m-1)ac}{m(a+c)} \wedge 4m=b \wedge \gcd(n,4m-1)\geq 1. \tag{2}$$

Let:

$$\{n\colon n=3+2k \wedge k \in [0,1,2,3,\dots]\}=\{3,5,7,\dots\}. \tag{3}$$

For all $n \in \{3,9,15,21,27,33,39,45,51,57,63,69,75,81,87,\dots\}$ and for some $a,c,b \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n}=\frac{1}{\frac{2n}{3}}+\frac{1}{n}+\frac{1}{\frac{2n}{3}} \wedge \frac{2n}{3}=a=c \wedge n=b\right].$$

On the strength of (3) $k \in \{0,3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,\dots\}$. For all n $\in$ $\{5,15,25,35,45,55,65,75,85,95,105,115,125\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$

$$\left[\frac{4}{n}=\frac{1}{\frac{4n}{5}}+\frac{1}{4n}+\frac{1}{\frac{2n}{5}} \wedge \frac{4n}{5}=a \wedge 4n=b \wedge \frac{2n}{5}=c\right].$$

On the strength of (3) $k \in \{1,6,11,16,21,26,31,36,41,46,51,56,61,66,71,76,81,86,91,96,\dots\}$.

For all $n \in \{5,11,17,23,29,35,41,47,53,59,65,71,77,83,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left|\frac{4}{n}=\frac{1}{\frac{n+1}{3}}+\frac{1}{n}+\frac{1}{\frac{n(n+1)}{3}} \wedge \frac{n+1}{3}=a \wedge n=b \wedge \frac{n(n+1)}{3}=c\right|.$$

On the strength of (3) $k \in \{1,4,7,10,13,16,19,22,25,28,31,34,37,40,43,46,49,52,55,58,61,\dots\}$.

On the strength of (2) for n=17 and for some some a,b,c ∈ {1,2,3,…}

$$nm=b \wedge m=1 \wedge a=c \wedge n=\frac{(4m-1)ac}{m(a+c)}=\frac{3ac}{a+c}\Big| \Rightarrow \left(n=17=\frac{3ac}{a+c} \wedge a=x+c\right) \Rightarrow 3c^2+(3x-34)c-17x=0 \Rightarrow$$

$\Delta=(3x)^2+34^2=290^2 \Rightarrow x=96.$

Therefore

$$\left(c=\frac{-254+290}{6}=6 \wedge a=96+c=102=6n \wedge 17c=6n\right).$$

Hence for all n ∈ {17,51,85,119,153,187,221,255,289,…} and some a,b,c ∈ {1,2,3,…}

$$\left|\frac{4}{n}=\frac{1}{6n}+\frac{1}{n}+\frac{1}{\frac{6n}{17}} \wedge 6n=a \wedge n=b \wedge \frac{6n}{17}=c\right|.$$

On the strength of (3) $k \in \{7,24,41,58,75,92,109,126,143,160,177,194,211,228,245,\dots\}$. For all n ∈ {3,7,11,15,19,23,27,31,335,39,43,47,51,55,…} and for some a,b,c ∈ {1,2,3,…}

$$\left|\frac{4}{n}=\frac{1}{\frac{n+1}{2}}+\frac{1}{\frac{n(n+1)}{4}}+\frac{1}{\frac{n+1}{2}} \wedge \frac{n+1}{2}=a=c \wedge \frac{n(n+1)}{4}=b\right|$$

On the strength of (3) $k \in \{0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,\dots\}$. For all n ∈ {7,21,35,49,63,77,91,105,119,133,147,161,…} and for some a,b,c ∈ {1,2,3,…}

$$\left|\frac{4}{n}=\frac{1}{\frac{4n}{7}}+\frac{1}{2n}+\frac{1}{\frac{4n}{7}} \wedge \frac{4n}{7}=a=c \wedge 2n=b\right|.$$

On the strength of (3) $k \in \{2,9,16,23,30,37,44,51,58,65,72,79,86,93,100,107,114,121,\dots\}$. For all n ∈ {11,33,55,77,99,121,143,165,187,209,231,…} and for some a,b,c ∈ {1,2,3,…}

$$\left|\frac{4}{n}=\frac{1}{\frac{6n}{11}}+\frac{1}{3n}+\frac{1}{\frac{6n}{11}} \wedge \frac{6n}{11}=a=c \wedge 3n=b\right|.$$

On the strength of (3) $k \in \{4,15,26,37,48,59,70,81,92,103,114,125,136,147,158,169,\dots\}$. For all n∈ {5,13,21,29,37,45,53,61,69,77,85,93,101,…} and for some a,b,c ∈ {1,2,3,…}

$$\left|\frac{4}{n}=\frac{1}{\frac{3n+1}{4}}+\frac{1}{\frac{n(3n+1)}{4}}+\frac{1}{\frac{3n+1}{8}} \wedge \frac{3n+1}{4}=a \wedge \frac{n(3n+1)}{4}=b \wedge \frac{3n+1}{8}=c\right|.$$

On the strength of (3) $k \in \{1,5,9,13,17,21,25,29,33,37,41,45,49,53,57,61,65,69,73,77,81,\dots\}$.

For all $n \in \{13,39,65,91,117,143,169,195,221,247,273,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left|\frac{4}{n}=\frac{1}{\frac{10n}{13}}+\frac{1}{10n}+\frac{1}{\frac{5n}{13}} \wedge \frac{10n}{13}=a \wedge 10n=b \wedge \frac{5n}{13}=c\right|.$$

On the strength of (3) $k \in \{5,18,31,44,57,70,83,96,109,122,135,148,161,174,187,200,\dots\}$.

On the strength of [1] for all $n \in \{13,23,33,43,53,63,73,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left|\frac{4}{n}=\frac{1}{\frac{3n+1}{10}}+\frac{1}{\frac{n(3n+1)}{4}}+\frac{1}{\frac{3n+1}{2}} \wedge \frac{3n+1}{10}=a \wedge \frac{n(3n+1)}{4}=b \wedge \frac{3n+1}{2}=c\right|.$$

On the strength of (3) $k \in \{5,10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,\dots\}$.

On the strength of [1] for all $n \in \{97,125,153,181,209,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left|\frac{4}{n}=\frac{1}{\frac{2(n+1)}{7}}+\frac{1}{2n}+\frac{1}{\frac{2n(n+1)}{7}} \wedge \frac{2(n+1)}{7}=a \wedge 2n=b \wedge \frac{2n(n+1)}{7}=c\right|.$$

On the strength of (3) $k \in \{47,61,75,89,103,117,131,145,159,173,187,201,215,229,243,\ldots\}$.
It is easy to verify that

$$\{0,3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,60,63,\ldots\} \cup$$
$$\{1,6,11,16,21,26,31,36,41,46,51,56,\ldots\} \cup \{1,4,7,10,13,16,19,22,25,28,31,34,37,\ldots\} \cup$$
$$\{7,24,41,58,75,92,109,126,143,160,177,\ldots\} \cup \{0,2,4,6,8,10,12,14,16,18,20,22,\ldots\} \cup$$
$$\{2,9,16,23,30,37,44,51,58,65,72,79,86,93,100,107,114,121,\ldots\} \cup$$
$$\{4,15,26,37,48,59,70,81,92,103,114,125,136,147,158,169,\ldots\} \cup$$
$$\{1,5,9,13,17,21,25,29,33,37,41,45,49,53,57,61,65,69,73,77,\ldots\} \cup$$
$$\{5,18,31,44,57,70,83,96,109,122,135,\ldots\} \cup \{5,10,15,20,25,30,35,\ldots\} \cup$$
$$\{47,61,75,89,103,117,131,145,159,173,187,201,215,229,243,\ldots\} = [0,1,2,3,\ldots].$$

Hence $n \in \{2,3,4,\ldots\}$, inasmuch as:

$$\{2\} \cup \{4,6,8,\ldots\} \cup \{3,9,15,\ldots\} \cup \{5,15,25,\ldots\} \cup \{5,11,17,\ldots\} \cup \{17,51,85,\ldots\} \cup$$
$$\{3,7,11,\ldots\} \cup \{7,21,35,\ldots\} \cup \{11,33,55,\ldots\} \cup \{5,13,21,\ldots\} \cup \{13,39,65,\ldots\} \cup$$
$$\{13,23,33,\ldots\} \cup \{97,125,153,\ldots\} = \{2,4,6,\ldots\} \cup \{3,5,7,\ldots\} = \{2,3,4,\ldots\}$$

Corollary 1. For all $n \in \{3,5,7,\ldots\}$ and for some $m,a,c,b \in \{1,2,3,\ldots\}$

$$\left[n = \frac{(4m-1)ac}{m(a+c)} \wedge nm = b \wedge \gcd(n,4m-1) \geq 1 \wedge (n \mid a,c,b \vee n \mid c,b \vee n \nmid b) \right].$$

This is the amazing proof. which was to be proved

References

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[2] http://en.wikipedia.org/wiki/Erd%C5%91s%E2%80%93Straus_conjecture

[3] http://www.cs.cmu.edu/~avelingk/papers/erdos_straus.pdf