Proof of the Beal Conjecture through the Fundamental Theorem of Arithmetic

Eric S. Watson

Abstract: This paper gives a proof of the Beal conjecture through the Fundamental Theorem of Arithmetic.

Theorem: If \(A^x + B^y = C^z\), where \(A, C, x, y, z\) are positive integers and \(x, y, z > 2\), then \(A, B, C\) must have a common prime factor.

Proof: According to the Fundamental Theorem of Arithmetic, every integer greater than 1 is either a prime itself or is the product of prime numbers. This means that \(A, C\) are either a prime itself or is the product of prime numbers. Since \(A, C\) are raised to exponential powers greater than 2 that means that \(A, B, C\) are going to be the products of prime numbers themselves or the products of other numbers that can be broken down as the product of primes. \(A, B, C\) cannot be equal to 1 because 1 is not a prime number, and 1 will always be 1 no matter what value of an exponent you raise it to. This means that the lowest common prime factor that \(A, B, C\) can have is 2.

1. Starting off with the Fundamental Theorem of Arithmetic we are told that every integer greater than 1 is either a prime number itself or is the product of prime numbers. [1]
2. Prime numbers are defined as positive integers that are divisible only by themselves and 1 only. By definition, this means that 2 is the smallest prime number. [1]
3. By [1] we can say that \(A, C\) are either prime numbers or the product of prime numbers.
4. The exponents \(x, y, z > 2\) so the values of these exponents can be any positive integer as long as it is greater than or equal to 3.
5. The exponents of a number say how many times to use the number in multiplication. This means that \(A^x\) says to multiply \(A\) by itself \(x\) times, \(B^y\) says to multiply \(B\) by itself \(y\) times, and \(C^z\) says to multiply \(C\) by itself \(z\) times.
6. With respect to 4 the minimum values that \(x, z\) can be is 3.
7. With respect to 2 the minimum values that \(A, C\) can be is 2.
8. The common prime factors that can exist between the positive integers of \(A, C\) in the equation \(A^x + B^y = C^z\) and \(x, y, z > 2\)
9. \(A, B, C\) must have a common prime factor. According to 2 the lowest common prime factor that can exist is 2.

References:

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