Recent Research Discourses On $\mathcal{K}$ -T$_{1/2}$ Spaces ($\mathcal{K}$ = α,p,β,s& pr)

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Keywords : $\mathcal{K}$ –T$_{1/2}$ spaces (where $\mathcal{K}$ = α, p,β,s & pr).

Abstract: Framing of this paper is to project the recent research work on $\mathcal{K}$ –T$_{1/2}$ spaces where $\mathcal{K}$ = α, p,β,s & pr and to overview their fundamental properties at a glance. However, the structure and characteristics possessed by these spaces are not always that easy to comprehend. These spaces are very useful in the study of certain objects in digital topology viz E.D.Khalinmsky et.al[23] showed that the digital line is a typical example of a T$_{1/2}$-space in which the closed sets and g-closed sets coincide.

1. Introduction & Preliminaries :

Being one among the important and interesting concepts in Topology, separation axioms are used to coin restricted classes of topological spaces as $\mathcal{K}$ –T$_{1/2}$ spaces where $\mathcal{K}$= α, p,β,s& pr(i.e. pre regular).

These $\mathcal{K}$ –T$_{1/2}$ spaces appear as the generalizations of specific closed sets. The first step of generalized closed sets was done by Levine[4] in 1970 and he initiated the notion T$_{1/2}$-space in unital topology which is properly placed between T$_0$ –space & T$_1$ –space.

After the works of Levine on semi open sets [3], several mathematicians turned their attention to the generalizations of various concepts of topology by considering semi-open sets instead of open sets. Bhattacharya & Lahiri [7] introduced and studied gs-closed as well as sg-closed sets which are weaker than g-closed sets. Maki et. al [9] &[10] introduced α- generalized closed & generalized α-closed sets (symbolized as αg- closed & gα-closed resp.)

The concept of a preopen set was introduced by Corson and Michael[1] who used the term "Locally Dense" and in [1a] & [2 a] these sets were called preopen sets.

Balachandran & Arockia Rani[11] defined the notion of generalized preclosed (i.e. gp-closed) sets. It follows from definitions that every αg-closed set is a gp-closed set. In 1997, Y. Gnanambal[17] conceptualized and studied generalized preregular closed (Symbolized as gpr-closed) sets.

The notions for closed sets to be generalized were defined using the closure operator (cl) & the interior operator(int)of a topological space(X,T) on which no separation axioms are assumed unless otherwise mentioned in the following manner:

Definition(1.1):

A subset A of a topological space (X,T) is called

 I. a semi-closed[3] if int(cl(A)) $\subseteq$ A..

 II. a pre-closed[2] if cl(int(A)) $\subseteq$ A.

 III. an α-closed[5] set if cl(int(cl(A))) $\subseteq$ A.

 IV. β-closed[6] if int(cl(int(A))) $\subseteq$ A.

 V. a regular closed[2] if A = cl(int(A)) and

 VI. a semi- regular[8] if A is both semi-open & semi-closed.

The complements of the above mentioned closed sets are the respective open sets.

Naturally, the semi-closure(resp. pre closure, α-closure ,β-closure & regular closure) of a subset A of a space (X,T) is the intersection of all semi-closed (resp. pre closed, α-closed, β-closed & regular closed) sets that contain A and is symbolized by scl(A) (resp. pcl(A), αcl(A), βcl(A) & rcl((A)).

Also, the semi-interior(resp. preinterior, α-interior ,β-interior & regular interior) of a subset A of a space (X,T) is the union of all semi-open (resp. pre open, α-open, β-open & regular open) sets that contain A and is symbolized by sint(A) (resp. pint(A), αint(A), βint(A) & rint((A)).

Definition (1.2): A subset A of a space (X,T) is said to be

(1) generalized closed (briefly g- closed)[4] set if cl(A)⊂U whenever A⊂ U & U is open in X .

(2) semi- generalized closed (briefly sg-closed) [4] set if scl(A) ⊂U whenever A ⊂ U & U is semi- open in X .

(3) generalized semi-closed (briefly gs-closed) [8] set if scl(A) ⊂U whenever A⊂ U & U is open in X .

(4) α- generalized closed (briefly αg-closed) [10] set if αcl(A) ⊂U whenever A⊂U & U is open in X.

(5) generalized α-closed (briefly gα-closed) [9] set if αcl(A) ⊂U whenever A⊂ U & U is α- open in X

(6) generalized pre-closed (briefly gp-closed) [17] set if pcl(A) ⊂U whenever A⊂ U & U is open in X .

(7) generalized semi-preclosed (briefly gβ-closed) [12] set if βcl(A) ⊂U whenever A⊂ U & U is β- open in X

(8) semi- pre generalized closed (briefly βg-closed) [12] set if βcl(A) ⊂U whenever A⊂ U & U is β - open in X.

(9) generalized preregular closed (briefly gpr-closed) [17] set if pcl(A) ⊂U whenever A⊂ U & U is regular- open in X.

(10) regular generalized closed (briefly rg-closed) [13] set if cl(A) ⊂U whenever A⊂ U & U is regular open in X.

In concluding this section let us recall the following concepts of different types of continuity and irresolute maps due to the sets mentioned in definitions (1.1) & (1.2) for their usefulness in the material of the preparation of this paper:

Definition (1.3):

A function f:(X,T) $\rightarrow$ (Y,σ) is said to be

(i)   semi-continuous (resp. irresolute)[3] if $f^{-1}(V)$ is semi-closed in (X,T) for every closed (resp. semi-closed) set V in (Y,σ),

(ii) pre-continuous (resp. pre- irresolute)[2 a] if $f^{-1}(V)$ is preclosed in (X,T) for every closed (resp. pre-closed) set V in (Y,σ),

(iii)α-continuous (resp.α- irresolute)[26] if $f^{-1}(V)$ is α-closed in (X,T) for every closed (resp. α-closed) set V in (Y,σ),

(iv)β-continuous (resp. β-irresolute)[27] if $f^{-1}(V)$ is β-closed in (X,T) for every closed (resp. β-closed) set V in (Y,σ),

(v) rg-continuous (resp. rg-irresolute)[13] if $f^{-1}(V)$ is rg-closed in (X,T) for every closed (resp. rg-closed) set V in (Y,σ),

(vi)regular-continuous (resp. regular irresolute)[13] if $f^{-1}(V)$ is regular closed in (X,T) for every closed (resp. regular closed) set V in (Y,σ),

(vii) g-continuous (resp. gc-irresolute)[28] if $f^{-1}(V)$ is g-closed in (X,T) for every closed (resp. g-closed) set V in (Y,σ),

(viii) sg-continuous (resp. sg-irresolute)[25] if $f^{-1}(V)$ is sg-closed in (X,T) for every closed(resp. sg-closed) set V in (Y,σ),

(ix) gs-continuous (resp. gs-irresolute)[25] if $f^{-1}(V)$ is gs-closed in (X,T) for every closed (resp. gs-closed) set V in (Y,σ),

(x) gα-continuous (resp. gα-irresolute)[9] if $f^{-1}(V)$ is gα-closed in (X,T) for every closed (resp. gα-closed) set V in (Y,σ),

(xi) αg-continuous (resp.αg- irresolute)[17] if $f^{-1}(V)$ is αg-closed in (X,T) for every open (resp. αg-closed) set V in (Y,σ),

(xii) gβ-continuous (resp. gβ irresolute)[12] if $f^{-1}(V)$ is gβ-closed in (X,T) for every closed (resp. gβ-closed) set V in (Y,σ),

(xiii) βg-continuous (resp.βg- irresolute)[12] if $f^{-1}(V)$ is βg-closed in (X,T) for every closed (resp. βg-closed) set V in (Y,σ),

(xiv) gp-continuous (resp. irresolute)[11] if $f^{-1}(V)$ is gp-closed in (X,T) for every closed (resp. gp-closed) set V in (Y,σ),

(xv) gpr-continuous (resp. gpr- irresolute)[3] if $f^{-1}(V)$ is gpr-closed in (X,T) for every closed (resp. gpr-closed) set V in (Y,σ),

Definition (1.3):

A function f:(X,T)$\rightarrow$(Y,σ) is said to be h-continuous (resp. h- irresolute)[3] if $f^{-1}(V)$ is h-closed in (X,T) for every closed (resp. h-closed) set V in (Y,σ),where h = semi, pre, α,β, rg, regular, g, sg, gs, gα, αg, gβ, βg , gp & gpr.([3],[2 a],[26],[27],[13],[28],[25],[9],[17],[12],[11] & [17]).

The following definition of different open & closed mappings play an important role in determining the topological property of on $\mathcal{K}$ –T$_{1/2}$ spaces under suitable & proper situations:

Definition (1.4):

A function f:(X,T)$\rightarrow$(Y,σ) is said to be h-open (resp. h- closed)[3] if f (U) is h-open (resp. h-closed) in (Y,σ) for every h-open(resp.h-closed) set U in (X,T), where h = pre, pre-semi, pre- α, pre-β, gp, sg, gs, gα, αg, gβ, βg ([15],[25],[21],[22],[27]&[24]).

§2. $\mathcal{K}$ –T$_{1/2}$ Spaces ($\mathcal{K}$ = α, p, β, s& pr):

In this section, the overview of the common facts possessed by $\mathcal{K}$ –T$_{1/2}$ spaces where $\mathcal{K}$= α, p,β,s& pr(i.e. pre regular) is presented at a glance for researchers. The literature survey on all on $\mathcal{K}$ –T$_{1/2}$ spaces(where $\mathcal{K}$= α, p,β,s& pr(i.e. pre regular) has been brought under a common framework.

Definition (2.1): $\mathcal{K}$ –T$_{1/2}$ spaces($\mathcal{K}$ =α,p,β) .

A topological space (X,T) is called $\mathcal{K}$ –T$_{1/2}$ space if every g $\mathcal{K}$ -closed set is $\mathcal{K}$-closed where $\mathcal{K}$= α, p,β([21],[2],[12,20] respectively).

Definition (2.2): Semi- T$_{1/2}$ space & Preregular - T$_{1/2}$ space:

(i) A topological space (X,T) is called semi-T$_{1/2}$ space (briefly s-T$_{1/2}$ space) if every sg-closed set is semi-closed [25].

(ii) A topological space (X,T) is called preregular -T$_{1/2}$ space (briefly pr-T$_{1/2}$ space) if every generalized pre regular closed (briefly gpr-closed) set is pre-closed[17].

The following definitions are enunciated for the existence of the forthcoming theorems:

Definition (2.3): In a topological space (X,T), the following notions are coined & well defined as:

(a) $\mathcal{K}$ D(X,T) = {A:A⊂X and A is g $\mathcal{K}$-closed in (X,T)}.

(b) $\mathcal{K}$ $cl^*(E)$ = ∩ {A:E ⊂ A ∈kD(X,T))}

(c) $\mathcal{K}O(X,T)^* = \{B:kcl^*(B^C)=B^C\}$. Here ($\mathcal{K}$ =α,p,β)

Theorem (2.4):

A topological space (X,T) is $\mathcal{K}$ –T$_{1/2}$ space if and only if $\mathcal{K}O(X,T)$ $\not\ni$ $O(X,T)^*$ where ($^\square$ $\mathcal{K}$ =α,p&β).

Proof: Necessity: Let a topological space(X,T) be $\mathcal{K}$-T$_{1/2}$ where ($\mathcal{K}$=α,p & β).This means $\mathcal{K}$-closed sets and g$\mathcal{K}$-closed sets coincide and so by the hypothesis $\mathcal{K}$cl(E)= $\mathcal{K}$cl*(E) holds good for every $\mathcal{K}$-closed subset E of (X,T). Hence, $\mathcal{K}$O(X,T)= $\mathcal{K}$O(X,T)* is in existence.

Sufficiency: Let A be a g$\mathcal{K}$-closed subset of(X,T).Then, we have $A=\mathcal{K}cl^*(A)$ & by the accepted criteria $\mathcal{K}O(X,T)= \mathcal{K}O(X,T)^*$, one can claim that $A^C \in \mathcal{K}O(X,T)$, which means that A is $\mathcal{K}$-closed. Therefore (X,T) fulfils the criteria for being $\mathcal{K}$-T$_{1/2}$ space.

Theorem (2.5). A topological space (X, T) is a $\mathcal{K}$ -T$_{1/2}$ space if and only if every singleton of X is either $\mathcal{K}$ -open or closed where ($\mathcal{K}$ = p & β).

Proof. Necessity: Let topological space (X, T) be a $\mathcal{K}$ - T$_{1/2}$ space where ($\mathcal{K}$ = p & β). Then every g $\mathcal{K}$ -closed set is $\mathcal{K}$ -closed.
Let us Suppose that for some x ∈ X such that {x} is not a closed set. This means that X − {x} is not an open set. So X is the only open set such that X − {x} ⊆ X.
Again, X − {x} ⊆X ⇒ $\mathcal{K}$cl(X −{x}) ⊆ $\mathcal{K}$cl(X) = X.
Thus, we have X − {x} ⊆X ⇒ $\mathcal{K}$cl(X −{x}) ⊆X & X is open.
Consequently X-{x} is a g$\mathcal{K}$ -closed set.
Since, (X,T) is $\mathcal{K}$ -T$_{1/2}$ space, hence, X −{x} turns to be $\mathcal{K}$ - closed or equivalently {x} is $\mathcal{K}$-open.

Sufficiency:

Let every singleton of X is either $\mathcal{K}$ -open or closed where ($\mathcal{K}$ =p &β). Let A ⊆X be a g$\mathcal{K}$ -closed set and x ∈X be such that x ∈ $\mathcal{K}$cl(A).

Case I. Let {x} be closed and x ∉ A. Then {x}⊆ $\mathcal{K}$cl(A) − A. But, we, however know that $\mathcal{K}$cl(A) − A does not contain any non -empty closed set for ($\mathcal{K}$ =p &β). This leads to the conclusion that x∈ A. So x ∈ $\mathcal{K}$cl(A) ⇒ x∈ A and $\mathcal{K}$cl(A) ⊆ A. But, in general A ⊆ $\mathcal{K}$cl(A). Consequently, A = $\mathcal{K}$cl(A) i.e. A is $\mathcal{K}$ − closed.

Case II. Let {x} is $\mathcal{K}$-open. Since, x ∈ $\mathcal{K}$cl(A), hence {x} ∩A≠φ implies that {x} ⊆ A. Therefore, $\mathcal{K}$cl(A) ⊆ A. Consequently, A = $\mathcal{K}$cl(A) i.e. A is $\mathcal{K}$ − closed.
Thus, whenever a singleton set is either closed or $\mathcal{K}$-open in (X,T) , it is observed that every g$\mathcal{K}$-closed set is $\mathcal{K}$ -closed. This leads to the conclusion that(X, T) is a $\mathcal{K}$ -T$_{1/2}$ space where ($\mathcal{K}$=p&β).

Hence, the theorem.

Theorem (2.6). A topological space (X, T) is $\mathcal{K}$ -T$_{1/2}$space if and only if every singleton of X is either $\mathcal{K}$ -open or $\mathcal{K}$ -closed where ($\mathcal{K}$ =α & s).

Proof. Necessity: Let topological space (X,T) be a $\mathcal{K}$ -T$_{1/2}$ space where ($\mathcal{K}$ =α&s). Then every $\wp$ - closed set is $\mathcal{K}$ –closed where $\mathcal{K}$ =α whenever $\wp$ = gα and $\mathcal{K}$ = s whenever $\wp$ = sg.

Let us suppose that for some $x \in$ X such that {x} is not a $\mathcal{K}$ closed set. This means that X –{x} is not $\mathcal{K}$ open set. So X is the only $\mathcal{K}$ -open set such that X – {x} $\subseteq$ X.

Again, X – {x} $\subseteq$ X $\Rightarrow$ $\mathcal{K}$cl(X –{x}) $\subseteq$ $\mathcal{K}$cl(X) = X.

Thus, we have X – {x} $\subseteq$ X $\Rightarrow$ $\mathcal{K}$cl(X –{x}) $\subseteq$ X & X is $\mathcal{K}$ -open.

Consequently X-{x} is a $\wp$-closed set.

Since, (X,T) is $\mathcal{K}$ -T$_{1/2}$ space, hence, X –{x} turns to be $\mathcal{K}$ - closed or equivalently {x} is $\mathcal{K}$- open.

Sufficiency:

Let every singleton of X be either $\mathcal{K}$ -open or $\mathcal{K}$ -closed where ($\mathcal{K}$ =α &s). Let A $\subseteq$X be a $\wp$- closed set where $\wp$ = gα whenever $\mathcal{K}$ =α and $\wp$ = sg whenever $\mathcal{K}$ = s .Let x $\in$X be such that x $\in$ $\mathcal{K}$cl(A).

Case I. Let {x} be $\mathcal{K}$ -closed and x$\notin$ A. Then {x}$\subseteq$ $\mathcal{K}$cl(A) − A. But, we, however know that $\mathcal{K}$cl(A) − A does not contain any non -empty $\mathcal{K}$ -closed set for ($\mathcal{K}$ =α &s). This leads to the conclusion that x$\in$ A. So x $\in$ $\mathcal{K}$ cl(A) $\Rightarrow$ x$\in$ A and $\mathcal{K}$cl(A) $\subseteq$ A. But, in general A $\subseteq$ $\mathcal{K}$cl(A). Consequently, A = $\mathcal{K}$cl(A) i.e. A is $\mathcal{K}$ – closed.

Case II. Let {x} be $\mathcal{K}$-open. Since, x $\in$ $\mathcal{K}$cl(A), hence {x} $\cap$A$\neq$φ implies that {x} $\subseteq$ A. Therefore, $\mathcal{K}$cl(A) $\subseteq$ A. Consequently, A = $\mathcal{K}$cl(A) i.e. A is $\mathcal{K}$ – closed.

Thus, whenever a singleton set is either $\mathcal{K}$ -closed or $\mathcal{K}$-open in (X,T) , it is observed that every $\wp$ - closed set is $\mathcal{K}$ –closed where $\wp$ = gα whenever $\mathcal{K}$ =α and $\wp$ = sg whenever $\mathcal{K}$ = s. This leads to the conclusion that(X, T) is a $\mathcal{K}$ -T$_{1/2}$ space where ($\mathcal{K}$ =α &s).
Hence, the theorem.

Theorem(2.7). A topological space (X, T) is preregular -T$_{1/2}$ (briefly pr-T$_{1/2}$) space if and only if every singleton of X is either preopen or regular closed .

Proof. Necessity: Let topological space (X, T) be a pr- T$_{1/2}$ space. Then every gpr-closed set is preclosed.

Let us Suppose that for some $x \in$ X such that {x} is not regular closed . This means that X –{x} is not regular open and X – {x} is trivially gpr-closed.
Since, (X,T) is pr -T$_{1/2}$ space, hence, X –{x} turns to be preclosed or equivalently, {x} is preopen.

Sufficiency:

Let A $\subseteq$X be a gpr -closed set . Let x $\in$ pcl(A). It is to show that x $\in$A. The following two cases are considered.

Case I. Let {x} be regular closed. Then if x$\notin$ A, there exists a regular closed set contained in pcl(A)– A. But pcl(A) – A does not contain any non -empty regular closed set, so x$\in$ A.

Case II. Let {x} be preopen. Since, $x \in$ pcl(A), hence, pcl(A) $\cap$ {x} $\neq \varphi$. Thus , $x \in$ A.

Now, in both the cases, $x \in$ A; which means that pcl(A) $\subseteq$ A or equivalently, A is preclosed. Consequently, (X,T) is a pr-$T_{1/2}$ space.

Hence, the theorem.

Theorem (2.8) :

(a)A space (X,T) is pr-$T_{1/2}$ space if and only if PO(T) = GPRO(T).

(b)Every pr-$T_{1/2}$ space is β-$T_{1/2}$ space but not the converse.

Proof: (a) ' if part' :

Let a space (X,T) be pr-$T_{1/2}$ space. Let A $\in$ GPRO(T). Then A^c is gpr-closed. By hypothesis, A^c is preclosed and so A $\in$ PO(T). Hence, GPRO(T) $\subseteq$ PO(T).

Again, A $\in$ PO(T) $\Rightarrow A^c \in$ PC(T)

$\qquad\qquad \Rightarrow A^c \in$ GPRC(T)

$\qquad\qquad \Rightarrow A \in$ GPRO(T)

i.e. $\qquad\qquad$ PO(T) $\subseteq$ GPRO(T)

Consequently, PO(T) = GPRO(T).

'only if part':

Let PO(T) = GPRO(T). Let A be gpr-closed. Then A^c is gpr-open. So $A^c \in$ PO(T). Equivalently A is preclosed, thereby applying (X,T) is pr-$T_{1/2}$ space.

(b) Let (X,T) be pr-$T_{1/2}$ space. Then by theorem (2.7) every singleton of (X,T) is closed or preopen . This implies that (X,T) is β- $T_{1/2}$ (J.Dontchev[12]).

The converse does not hold good.

Example (2.8) : Let X = {a,b,c} ,T = {φ, X, {a},{b},{a,b}}. Then every singleton of (X,T) is closed or preopen . Therefore, (X,T) is β- $T_{1/2}$ space but not pr-$T_{1/2}$ space since the gpr-closed set {a,b} is not pre-closed.

Hence, the theorem.

Theorem (2.9):

If f:(X,T) $\rightarrow$ (Y,σ) is onto , $g^{\mathcal{K}}$ –irresolute & $\mathcal{R}$-closed mapping , then [(X,T) is a $\mathcal{K}$ –$T_{1/2}$ space] $\Rightarrow$ [(Y,σ) is also a $\mathcal{K}$ –$T_{1/2}$ space], where $\mathcal{K}$ = p,α,β,pr according as $\mathcal{R}$ = pre, pre –α, pre-β & p.

Proof:

Let f:(X,T) $\rightarrow$ (Y,σ) be onto , $g^{\mathcal{K}}$ –irresolute & $\mathcal{R}$ -closed mapping where $\mathcal{K}$ = p,α,β,pr according as $\mathcal{R}$ = pre, pre –α, pre-β & p.

Let A be $g^{\mathcal{K}}$-closed set of (Y,σ).

Since, f is $g^{\mathcal{K}}$ –irresolute, hence, f^{-1}(A) is also a $g^{\mathcal{K}}$-closed set in (X,T).

Since, (X,T) is a $\mathcal{K}$ –$T_{1/2}$ space, hence, f^{-1}(A) is also a $\mathcal{K}$ -closed set in (X,T).

Since, f is $\mathcal{R}$ -closed & onto mapping, hence, $f(f^{-1}(A))$ = A must be a $\mathcal{K}$ -closed set in (Y,σ).

Consequently, every $g^{\mathcal{K}}$ -closed set of (Y,σ) is a $\mathcal{K}$ -closed set .i.e. (Y,σ) is a $\mathcal{K}$ –$T_{1/2}$ space.

Thus , [(X,T) is a $\mathcal{K}$ –T $_{1/2}$ space] $\Rightarrow$ [(Y,σ) is also a $\mathcal{K}$ –T $_{1/2}$ space].
Hence, the theorem.

Conclusion :

The recent research discourses on $\mathcal{K}$ –T $_{1/2}$ spaces where $\mathcal{K}$ = α, p,β,s & pr provide the development of different T $_{1/2}$ spaces over the period of time. The literature survey on $\mathcal{K}$ –T $_{1/2}$ spaces is an extension of Levine's definition of the T $_{1/2}$ space in which the closed sets and g-closed sets coincide. This paper also includes the characterisation of $\mathcal{K}$ –T $_{1/2}$ spaces through singleton set by stating that every singleton of X is either $\mathcal{K}$ -open or $\mathcal{K}$ -closed / closed which is the motivation by Dunham's characterisation of T $_{1/2}$ spaces through singletons by defining that every singleton of X is either open or closed (1977). Also, the contents of this paper are very useful for the researchers to overview these $\mathcal{K}$ –T $_{1/2}$ spaces and their topological property at one place and at a glance.

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