

## THE PROOF OF THE FERMAT-BEAL THEOREM

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**Dedicatory** Dedicated to my Family

**Abstract:** The proof of The Beal's Conjecture.

### I. INTRODUCTION

The famous Fermat's Last Theorem (FLT) assertion that for  $n \in \{3, 4, 5, \dots\}$  and for all  $X, Y, Z \in \{1, 2, 3, \dots\}$ :  $X^n + Y^n \neq Z^n \in \mathbb{1}$ .

It is easy to see that if  $X^n + Y^n = Z^n$  then either  $X, Y, Z$  are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor).

You could then restate FLT by saying that  $X^n + Y^n = Z^n$  is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [1]

### II. THE FERMAT-BEAL THEOREM

Theorem (Fermat- Beal Theorem) For all  $n \in \{3, 4, 5, \dots\}$  the equation  $A^n + B^n = C^n$  has no primitive solutions in  $\{1, 2, 3, \dots\}$ .

**Proof.** Suppose that for some  $x, y, z \in \{3, 4, 5, \dots\}$  the equation

$$A^n + B^n = C^n$$

has primitive solutions in  $\{1, 2, 3, \dots\}$ .

Thus for some  $x, y, z \in \{3, 4, 5, \dots\}$  and for some coprime  $A, B, C \in \{1, 2, 3, \dots\}$  and for some coprime  $a, b, c \in \{1, 2, 3, \dots\}$  such that only one number out of  $(A, B, C)$  is even :

$$[A^n + (B^n)^{\frac{z}{c}} = (C^n)^{\frac{z}{c}} \wedge x | y, z \wedge x \neq z] \vee$$

$$[(A^n)^{\frac{z}{c}} + B^n = (C^n)^{\frac{z}{c}} \wedge z \wedge x \neq z] \vee$$

$$[(A^n)^{\frac{z}{c}} + (B^n)^{\frac{z}{c}} = (C^n)^{\frac{z}{c}} \wedge z | x, y \wedge x \neq y] \vee$$

$$[A^n + B^n = C^n \wedge x = y = z] \vee$$

$$[A^n + (B^n)^{\frac{z}{c}} = (C^n)^{\frac{z}{c}} \wedge x \nmid y, z \wedge b^z = B \wedge c^z = C \wedge$$

$$A^n + (b^z)^{\frac{z}{c}} = (c^z)^{\frac{z}{c}} \wedge \gcd(A, b) = \gcd(A, c) = 1] \vee$$

$$[(A^n)^{\frac{z}{c}} + B^n = (C^n)^{\frac{z}{c}} \wedge y \nmid x, z \wedge a^z = A \wedge c^z = C \wedge$$

$$(a^z)^{\frac{z}{c}} + B^n = (c^z)^{\frac{z}{c}} \wedge \gcd(a, B) = \gcd(B, c) = 1] \vee$$

$$[(A^n)^{\frac{z}{c}} + (B^n)^{\frac{z}{c}} = C^n \wedge z \nmid x, y \wedge a^z = A \wedge b^z = B \wedge$$

$$(a^z)^x + (b^y)^z = C^x \wedge \text{gcd}(a,C) = \text{gcd}(b,C) = 1] \vee$$

$$[A^z + (B^y)^z = (C^x)^z \wedge x|y \wedge x \nmid z \wedge y \neq z \wedge c^z = C \wedge$$

$$A^z + (B^y)^z = (c^x)^z \wedge \text{gcd}(A,c) = \text{gcd}(B,c) = 1] \vee$$

$$[A^z + (B^y)^z = (C^x)^z \wedge x|z \wedge x \nmid y \wedge y \neq z \wedge b^z = B \wedge$$

$$A^z + (b^y)^z = (C^x)^z \wedge \text{gcd}(A,b) = \text{gcd}(b,C) = 1] \vee$$

$$[(A^z)^y + B^y = (C^x)^y \wedge y|x \wedge y \nmid z \wedge x \neq z \wedge c^y = C \wedge$$

$$(A^z)^y + B^y = (c^x)^y \wedge \text{gcd}(A,c) = \text{gcd}(B,c) = 1] \vee$$

$$[(A^z)^y + B^y = (C^x)^y \wedge y|z \wedge y \nmid x \wedge x \neq z \wedge a^y = A \wedge$$

$$(a^z)^y + B^y = (C^x)^y \wedge \text{gcd}(a,B) = \text{gcd}(a,C) = 1] \vee$$

$$[(A^z)^z + (B^y)^z = C^z \wedge z|x \wedge z \nmid y \wedge x \neq y \wedge b^z = B \wedge$$

$$(A^z)^z + (b^y)^z = C^z \wedge \text{gcd}(A,b) = \text{gcd}(b,C) = 1] \vee$$

$$[(A^z)^z + (B^y)^z = C^z \wedge z|y \wedge z \nmid x \wedge x \neq y \wedge a^z = A \wedge$$

$$(a^z)^z + (B^y)^z = C^z \wedge \text{gcd}(a,B) = \text{gcd}(a,C) = 1],$$

which is false because FLT is true. ♠

The Last Case For some  $x, y, z \in \{3,4,5, \dots\}$  and for some coprime  $X, Y, Z \in \{1,2,3, \dots\}$  and for some coprime  $a, b, c \in \{1,2,3, \dots\}$  such that only one number out of  $(a, b, c)$  is even :

$$[(X = a^z \wedge Y = b^y \wedge Z = c^z \wedge a^{xz} + b^{yz} = c^{yz} \equiv 0)$$

Thus for some  $x, y, z \in \{3,4,5, \dots\}$  and for some coprime  $a, b, c \in \{1,2,3, \dots\}$  and for some coprime  $A, B, C \in \{1,2,3, \dots\}$  such that only one number out of  $(A, B, C)$  is even :

$$[(a^{xz} + b^{yz} = c^{yz} = A^z + B^y = C^z \equiv 0 \wedge a^z = A \wedge b^y = B \wedge c^z = C) \in 0],$$

inasmuch as the Fermat Equation is false. This is the proof.

**REFERENCES**

[1] <http://www.ijetconjecture.com>  
 [2] Gula, W.: [http://www.ijetae.com/files/Volume2Issue12/IJETAE\\_1212\\_14.pdf](http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf)