

THE PROOF OF THE FERMAT-BEAL THEOREM

Leszek W. Guła

Lublin-POLAND

Keywords: Beal Conjecture, Diophantine Equations, Fermat Theorem, Greatest Common Divisor, Number theory.

Dedicatory Dedicated to my Family

Abstract: The proof of The Beal's Conjecture.

I. INTRODUCTION

The famous Fermat's Last Theorem (FLT) assertion that for $n \in \{3, 4, 5, \dots\}$ and for all $X, Y, Z \in \{1, 2, 3, \dots\}$: $X^n + Y^n \neq Z^n \in \mathbb{1}$.

It is easy to see that if $X^n + Y^n = Z^n$ then either X, Y, Z are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor).

You could then restate FLT by saying that $X^n + Y^n = Z^n$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [1]

II. THE FERMAT-BEAL THEOREM

Theorem (Fermat- Beal Theorem) For all $n \in \{3, 4, 5, \dots\}$ the equation $A^n + B^n = C^n$ has no primitive solutions in $\{1, 2, 3, \dots\}$.

Proof. Suppose that for some $x, y, z \in \{3, 4, 5, \dots\}$ the equation

$$A^n + B^n = C^n$$

has primitive solutions in $\{1, 2, 3, \dots\}$.

Thus for some $x, y, z \in \{3, 4, 5, \dots\}$ and for some coprime $A, B, C \in \{1, 2, 3, \dots\}$ and for some coprime $a, b, c \in \{1, 2, 3, \dots\}$ such that only one number out of (A, B, C) is even :

$$[A^n + (B^n)^x = (C^n)^x \wedge x | y, z \wedge x \neq z] \vee$$

$$[(A^n)^y + B^n = (C^n)^y \wedge y | x, z \wedge x \neq z] \vee$$

$$[(A^n)^x + (B^n)^z = (C^n)^z \wedge z | x, y \wedge x \neq y] \vee$$

$$[A^n + B^n = C^n \wedge x = y = z] \vee$$

$$[A^n + (B^n)^x = (C^n)^x \wedge x | y, z \wedge b^x = B \wedge c^x = C \wedge$$

$$A^n + (b^y)^x = (c^z)^x \wedge \gcd(A, b) = \gcd(A, c) = 1] \vee$$

$$[(A^n)^y + B^n = (C^n)^y \wedge y | x, z \wedge a^y = A \wedge c^y = C \wedge$$

$$(a^x)^y + B^n = (c^z)^y \wedge \gcd(a, B) = \gcd(B, c) = 1] \vee$$

$$[(A^n)^x + (B^n)^z = (C^n)^z \wedge z | x, y \wedge a^z = A \wedge b^z = B \wedge$$

$$(a^z)^x + (b^y)^z = C^x \wedge \text{gcd}(a,C) = \text{gcd}(b,C) = 1] \vee$$

$$[A^z + (B^y)^z = (C^x)^z \wedge x|y \wedge x \nmid z \wedge y \neq z \wedge c^z = C \wedge$$

$$A^z + (B^y)^z = (c^x)^z \wedge \text{gcd}(A,c) = \text{gcd}(B,c) = 1] \vee$$

$$[A^z + (B^y)^z = (C^x)^z \wedge x|z \wedge x \nmid y \wedge y \neq z \wedge b^z = B \wedge$$

$$A^z + (b^y)^z = (C^x)^z \wedge \text{gcd}(A,b) = \text{gcd}(b,C) = 1] \vee$$

$$[(A^z)^y + B^y = (C^x)^y \wedge y|x \wedge y \nmid z \wedge x \neq z \wedge c^y = C \wedge$$

$$(A^z)^y + B^y = (c^x)^y \wedge \text{gcd}(A,c) = \text{gcd}(B,c) = 1] \vee$$

$$[(A^z)^y + B^y = (C^x)^y \wedge y|z \wedge y \nmid x \wedge x \neq z \wedge a^y = A \wedge$$

$$(a^z)^y + B^y = (C^x)^y \wedge \text{gcd}(a,B) = \text{gcd}(a,C) = 1] \vee$$

$$[(A^z)^z + (B^y)^z = C^z \wedge z|x \wedge z \nmid y \wedge x \neq y \wedge b^z = B \wedge$$

$$(A^z)^z + (b^y)^z = C^z \wedge \text{gcd}(A,b) = \text{gcd}(b,C) = 1] \vee$$

$$[(A^z)^z + (B^y)^z = C^z \wedge z|y \wedge z \nmid x \wedge x \neq y \wedge a^z = A \wedge$$

$$(a^z)^z + (B^y)^z = C^z \wedge \text{gcd}(a,B) = \text{gcd}(a,C) = 1],$$

which is false because FLT is true. ♠

The Last Case For some $x, y, z \in \{3,4,5, \dots\}$ and for some coprime $X, Y, Z \in \{1,2,3, \dots\}$ and for some coprime $a, b, c \in \{1,2,3, \dots\}$ such that only one number out of (a, b, c) is even :

$$[(X = a^z \wedge Y = b^y \wedge Z = c^z \wedge a^{zx} + b^{zy} = c^{zx} \equiv 0)$$

Thus for some $x, y, z \in \{3,4,5, \dots\}$ and for some coprime $a, b, c \in \{1,2,3, \dots\}$ and for some coprime $A, B, C \in \{1,2,3, \dots\}$ such that only one number out of (A, B, C) is even :

$$[(a^{zx} + b^{zy} = c^{zx} = A^z + B^y = C^z \equiv 0 \wedge a^z = A \wedge b^y = B \wedge c^z = C) \in 0],$$

inasmuch as the Fermat Equation is false. This is the proof.

REFERENCES

[1] <http://www.ijetconjecture.com>
 [2] Gula, W.: http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf