THE PROOF OF THE FERMAT-BEAL THEOREM

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Abstract: The proof of The Beal's Conjecture.

I. INTRODUCTION

The famous Fermat's Last Theorem (FLT) assertion that for all $x \in \{3, 4, 5, \dots\}$ and for all $X, Y, Z \in \{1, 2, 3, \dots\}$: $X^x + Y^x \neq Z^x \in 1$.

It is easy to see that if $X^x + Y^x = Z^x$ then either X, Y, Z are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor).

You could then restate FLT by saying that $X^x + Y^x = Z^x$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [1]

II. THE FERMAT-BEAL THEOREM

Theoreme (Fermat- Beal Theorem) For all $x \in \{3, 4, 5, \dots\}$ the equation $A^x + B^x = C^x$ has no primitive solutions in $\{1, 2, 3, \dots\}$.

Proof. Suppose that for some $x, y, z \in \{3, 4, 5, \dots\}$ the equation

$$A^x + B^x = C^x$$

has primitive solutions in $\{1, 2, 3, \dots\}$.

Thus for some $x, y, z \in \{3, 4, 5, \dots\}$ and for some coprime $A, B, C \in \{1, 2, 3, \dots\}$ and for some coprime $a, b, c \in \{1, 2, 3, \dots\}$ such that only one number out of (A, B, C) is even :

$$\left[A^x + \left(\frac{B^x}{c^x} \right) = \left(\frac{C^x}{c^x} \right) \wedge x \mid y, z \wedge x \neq z \right] \vee$$

$$\left[\left(\frac{A^x}{c^x} \right) + B^x = \left(\frac{C^x}{c^x} \right) \wedge x \neq z \wedge x \neq y \right] \vee$$

$$\left[\left(\frac{A^x}{c^x} \right) + \left(\frac{B^x}{c^x} \right) = \left(\frac{C^x}{c^x} \right) \wedge x \mid x, y \wedge x \neq y \right] \vee$$

$$[A^x + B^x = C^x \wedge x = y = z] \vee$$

$$[A^x + \left(\frac{B^x}{c^x} \right) = \left(\frac{C^x}{c^x} \right) \wedge x \nmid y, z \wedge b^x = B \wedge c^x = C \wedge$$

$$A^x + (b^x)^x = (c^x)^x \wedge \gcd(A, b) = \gcd(A, c) = 1] \vee$$

$$\left[\left(\frac{A^x}{c^x} \right) + B^x = \left(\frac{C^x}{c^x} \right) \wedge y \nmid x, z \wedge a^x = A \wedge c^x = C \wedge$$

$$(a^x)^x + B^x = (c^x)^x \wedge \gcd(a, B) = \gcd(B, c) = 1] \vee$$

$$\left[\left(\frac{A^x}{c^x} \right) + \left(\frac{B^x}{c^x} \right) = \left(\frac{C^x}{c^x} \right) \wedge z \nmid x, y \wedge a^x = A \wedge b^x = B \wedge$$

$$(a^x)^z + (b^y)^z = c^z \wedge \gcd(a, c) = \gcd(b, c) = 1] \vee$$

$$[A^x + \left(\frac{y}{Bz}\right)^z = \left(\frac{z}{Cz}\right)^z \wedge x | y \wedge x \nmid z \wedge y \neq z \wedge c^z = C \wedge$$

$$A^x + \left(\frac{y}{Bz}\right)^z = (c^z)^z \wedge \gcd(A, c) = \gcd(B, c) = 1] \vee$$

$$[A^x + \left(\frac{y}{Bz}\right)^z = \left(\frac{z}{Cz}\right)^z \wedge x | z \wedge x \nmid y \wedge y \neq z \wedge b^z = B \wedge$$

$$A^x + (b^y)^z = \left(\frac{z}{Cz}\right)^z \wedge \gcd(A, b) = \gcd(b, C) = 1] \vee$$

$$[(\frac{z}{Az})^y + B^y = \left(\frac{z}{Cz}\right)^y \wedge y | x \wedge y \nmid z \wedge x \neq z \wedge c^y = C \wedge$$

$$\left(\frac{z}{Az}\right)^y + B^y = (c^y)^y \wedge \gcd(A, c) = \gcd(B, c) = 1] \vee$$

$$[(\frac{z}{Az})^y + B^y = \left(\frac{z}{Cz}\right)^y \wedge y | z \wedge y \nmid x \wedge x \neq z \wedge a^y = A \wedge$$

$$(a^x)^y + B^y = \left(\frac{z}{Cz}\right)^y \wedge \gcd(a, B) = \gcd(a, C) = 1] \vee$$

$$[(\frac{z}{Az})^z + \left(\frac{y}{Bz}\right)^z = C^z \wedge z | x \wedge z \nmid y \wedge x \neq y \wedge b^z = B \wedge$$

$$\left(\frac{z}{Az}\right)^z + (b^y)^z = C^z \wedge \gcd(A, b) = \gcd(b, C) = 1] \vee$$

$$[(\frac{z}{Az})^z + \left(\frac{y}{Bz}\right)^z = C^z \wedge z | y \wedge z \nmid x \wedge x \neq y \wedge a^z = A \wedge$$

$$(a^x)^z + \left(\frac{y}{Bz}\right)^z = C^z \wedge \gcd(a, B) = \gcd(a, C) = 1]],$$

which is false because FLT is true. ♠

The Last Case For some $x, y, z \in \{3, 4, 5, \dots\}$ and for some coprime $X, Y, Z \in \{1, 2, 3, \dots\}$ and for some coprime $a, b, c \in \{1, 2, 3, \dots\}$ such that only one number out of (a, b, c) is even :

$$[(X = a^x \wedge Y = b^y \wedge Z = c^z \wedge a^{xz} + b^{yz} = c^{xz} \equiv 0)]$$

Thus for some $x, y, z \in \{3, 4, 5, \dots\}$ and for some coprime $a, b, c \in \{1, 2, 3, \dots\}$ and for some coprime $A, B, C \in \{1, 2, 3, \dots\}$ such that only one number out of (A, B, C) is even :

$$[(a^{xz} + b^{yz} = c^{xz} = A^x + B^y = C^z \equiv 0 \wedge a^x = A \wedge b^y = B \wedge c^z = C) \in 0],$$

inasmuch as the Fermat Equation is false. This is the proof.

REFERENCES

- [1] <http://www.ijetconjecture.com>
- [2] Gula, W.: http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf