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INDEX SUMMABILITY OF AN INFINITE SERIES USING δ-QUASI MONOTONE SEQUENCE

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ABSTRACT: A result concerning absolute indexed Summability factor of an infinite series using δ - Quasi monotone sequence has been established.

1. INTRODUCTION:

A positive sequence (a_n) is said to be almost increasing if there exists a positive sequence (b_n) and two positive constants A and B such that

$$Ab_n \le a_n \le Bb_n \text{ , for all } n . \tag{1.1}$$

The sequence (a_n) is said to be quasi- β -power increasing, if there exists a constant K depending upon β with $K \geq 1$ such that

$$K n^{\beta} a_n \ge m^{\beta} a_m \ , \tag{1.2}$$

for all n≥m . In particular, if $\beta = 0$, then (a_n) is said to be quasi-increasing sequence. It is clear thatevery almost increasing sequence is a quasi- β-power increasing sequence for any non-negative β.But the converse is not true as $(n^{-\beta})$ is quasi- β-power increasing but not almost increasing.
Let $f = (f_n)$ be a positive sequence of numbers. Then the positive sequence (a_n) is said to be quasi-f-power increasing, if there exists a constant K depending upon f with K ≥1such that

$$K f_n a_n \ge f_m a_m , \tag{1.3}$$

for n≥m ≥1([6]) . Clearly, if (a_n) is a quasi- f-power increasing sequence, then the ($a_n f_n$)is a quasi-increasing sequence.

Let $\delta = (\delta_n)$ be a positive sequence of numbers. Then a positive sequence (a_n) is said to be δ-quasi monotone [1], if $a_n \to 0$, $a_n > 0$ ultimately and $\Delta d_n \geq -\delta_n$, where $\Delta d_n = d_n - d_{n+1}$.
Let $\sum a_n$ be an infinite series with sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{n} p_v \to \infty, \text{ as } n \to \infty$$

Then the sequence-to-sequence transformation

$$T_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v , \; P_n \neq 0, \tag{1.4}$$

defines the $(\bar{N}, p_n)$ mean of the sequence (s_n) generated by the sequence of coefficients $\{p_n\}$. The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n\right|_k, k \geq 1([2])$, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} \left|T_n - T_{n-1}\right|^k < \infty . \tag{1.5}$$

The series $\sum a_n$ is said to be summable $\left|\bar{N}, p_n ; \delta\right|_k, k \geq 1, \delta \geq 0$, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{\delta k + k - 1} \left|T_n - T_{n-1}\right|^k < \infty . \tag{1.6}$$

2. KNOWN THEOREMS:

Dealing with quasi- β-power increasing sequence Bor and Debnath **[3]** have established the following theorem:

2.1. THEOREM:

Let (X_n) be a quasi-β-power increasing sequence for $0 < \beta < 1$ and (λ_n) be a real sequence.
If the conditions

$$\sum_{n=1}^{m} \frac{P_n}{n} = O(P_m), \tag{2.1.1}$$

$$\lambda_n X_n = O(1), \tag{2.1.2}$$

$$\sum_{n=1}^{m} \frac{|t_n|^k}{n} = O(X_m), \tag{2.1.3}$$

$$\sum_{n=1}^{m} \frac{p_n |t_n|^k}{P_n} = O(X_m) \tag{2.1.4}$$

and

$$\sum_{n=1}^{m} nX_n |\Delta^2 \lambda_n| < \infty \tag{2.1.5}$$

are satisfied, where t_n is the (C,1) mean of the sequence (na_n) .Then the series $\sum a_n \lambda_n$ is summable $|\overline{N}, p_n|_k, k \geq 1$

Subsequently Leindler [4] established a similar result reducing certain condition of Bor. He established:

2.2. THEOREM:

Let the sequence (X_n) be a quasi-β-power increasing sequence for $0 < \beta < 1$, and the real sequence (λ_n) satisfies the conditions

$$\sum_{n=1}^{m} \lambda_n = O(m) \tag{2.2.2}$$

and

$$\sum_{n=1}^{m} |\Delta \lambda_m| = O(m) . \tag{2.2.3}$$

hold, where $X_n(\beta) = \max\left(n^{\beta} X_n, \log n\right)$.Then the series $\sum a_n \lambda_n$ is summable $|\overline{N}, p_n|_k, k \geq 1$.

Recently, extending the above results to quasi-f-power increasing sequence, Sulaiman **[7]** have established the following theorem:

2.3. THEOREM:

Let $f = (f_n) = (n^{\beta} \log^{\gamma} n), 0 \leq \beta < 1, \gamma \geq 0$ be a sequence. Let (X_n) be a quasi-f–power sequence and (λ_n) a sequence of constants satisfying the conditions

$$\lambda_n \to 0 \ as\, n \to \infty, \tag{2.3.1}$$

$$\sum_{n=1}^{\infty} nX_n |\Delta|\Delta \lambda_n|| < \infty, \tag{2.3.2}$$

$$|\lambda_n| X_n = O(1), \tag{2.3.3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n X_n^{k-1}} |t_n|^k = O(X_m) \tag{2.3.4}$$

and

$$\sum_{n=1}^{\infty} \frac{p_n}{P_n} \frac{1}{X_n^{k-1}} |t_n|^k = O(X_m), \tag{2.3.5}$$

where t_n is the (C,1)mean of the sequence (na_n) .Then the series $\sum a_n \lambda_n$ is summable $|\overline{N}, p_n|_k, k \geq 1$.

Very recently, Misra et al [5] established the following theorem:

2.4. THEOREM:

Let $f = (f_n) = (n^{\beta}(\log n)^{\gamma})$ be a sequence and (X_n) be a quasi- f -power sequence. Let (λ_n) a sequence of constants such that

$$\lambda_n \to 0, \ as \ n \to \infty, \tag{2.4.1}$$

$$\sum_{n=1}^{\infty} n X_n |\Delta|\Delta\lambda_n|| < \infty, \tag{2.4.2}$$

$$|\lambda_n| X_n = O(1), \tag{2.4.3}$$

$$\sum_{n=v+1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k-1} \frac{1}{P_{n-1}} = O\left(\frac{P_m}{p_m}\right)^{\delta k-1}, \tag{2.4.4}$$

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k-1} \frac{|t_v|^k}{X_v^{k-1}} = O(X_m), \tag{2.4.5}$$

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{\delta k} \frac{|t_n|^k}{n X_n^{k-1}} = O(X_m). \tag{2.4.6}$$

Then the series $\sum a_n \lambda_n$ is summable $|\overline{N}, p_n; \delta|_k, k \geq 1, \delta \geq 0.$

In what follows in this paper, using δ - quasi monotone sequence, we prove the following theorem.

3. MAIN THEOREM

Let $f=(f_n)=\left(n^{\beta}(\log n)^{\gamma}\right)$ be a sequence and (X_n) be a quasi-f-power sequence. Suppose also that there exists a sequence of numbers (A_n) such that it is δ – quasi – monotone with

$$\sum n\delta_n X_n < \infty \tag{3.1}$$

$$\Delta A_n \le \delta_n \quad \text{for all } n. \tag{3.2}$$

Let (λ_n) a sequence of constants such that

$$\lambda_n \to 0,\ as\ \ n \to \infty, \tag{3.3}$$

$$|\lambda_n| X_n = O(1), \tag{3.4}$$

and

$$|\Delta\lambda_n| \le |A_n| \quad \text{for all } \ n. \tag{3.5}$$

Then the series $\sum a_n\lambda_n$ is summable $\left|\overline{N}, p_n; \sigma\right|_k, k \ge 1, \sigma \ge 0.$, if

$$\sum_{n=v+1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k-1}\frac{1}{P_{n-1}} = O\left(\frac{P_v}{p_v}\right)^{\sigma k-1}, \tag{3.6}$$

$$\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k-1}\frac{|t_n|^k}{X_n^{k-1}} = O(X_m), \tag{3.7}$$

$$\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k}\frac{|t_n|^k}{nX_n^{k-1}} = O(X_m), \tag{3.8}$$

where (t_n) is the n th (C,1) mean of the sequence ($n\, a_n$).
In order to prove the theorem we require the following lemma.

4. LEMMA:

Let $f=(f_n)=\left(n^{\beta}(\log n)^{\gamma}\right), 0 \le \beta < 1, \gamma \ge 0$ be a sequence and (X_n) be a quasi - f – power increasing sequence. Let (A_n) be a sequence of numbers such that it is δ – quasi – monotone satisfying (3.1) and (3.2). then

$$nX_n|A_n| = O(1) \tag{4.1}$$

and

$$\sum_{n=1}^{m} X_n|A_n| < \infty, \text{as } m \to \infty. \tag{4.2}$$

4.1. PROOF OF THE LEMMA:

As $A_n \to 0$ and $n^\beta (\log n)^\gamma X_n$ is non-decreasing, we have

$$n X_n |A_n| = n^{1-\beta} (\log n)^{-\gamma} \left(n^\beta (\log n)^\gamma X_n\right) \sum_{\upsilon=n}^{\infty} \Delta |A_\upsilon|$$

$$= O(1) n^{1-\beta} (\log n)^\gamma n \sum_{\upsilon=n}^{\infty} \nu^\beta (\log \nu)^\gamma X_\upsilon |\Delta A_\upsilon|$$

$$= O(1) \sum_{\upsilon=n}^{\infty} \nu^{1-\beta} (\log \nu)^{-\gamma} \nu^\beta (\log \nu)^\gamma X_\upsilon |\Delta A_\upsilon|$$

$$= O(1) \sum_{\upsilon=n}^{\infty} \nu X_\upsilon |\Delta A_\upsilon| .$$

$$= O(1) \sum_{\upsilon=n}^{\infty} \nu X_\upsilon |\delta_\upsilon|$$

$$= O(1)$$

This establishes (4.1).

Next

$$\sum_{n=1}^{m} X_n |A_n| = \sum_{n=1}^{m-1} \left(\sum_{r=1}^{n} X_r \right) \Delta |A_n| + |A_m| \left(\sum_{r=1}^{m} X_r \right)$$

$$= O(1) \sum_{n=1}^{m-1} \left(\sum_{r=1}^{n} r^{-\beta} (\log r)^{-\gamma} r^\beta (\log r)^\gamma X_r \right) \Delta |A_n|$$

$$+ O(1) \left(\sum_{r=1}^{m} r^{-\beta} (\log r)^{-\gamma} r^\beta (\log r)^\gamma X_r \right) |A_m|$$

$$= O(1) \sum_{n=1}^{m-1} \left(n^\beta (\log n)^\gamma X_n\right) \Delta |A_n| \sum_{r=1}^{n} r^{-\beta-\epsilon} (\log r)^{-\gamma} r^\epsilon$$

$$+ O(1) m^\beta X_m |A_m| (\log m)^\gamma \sum_{r=1}^{m} r^{-\beta-\epsilon} (\log r)^{-\gamma} r^\epsilon, \epsilon < 1 - \beta.$$

$$= O(1) \sum_{n=1}^{m-1} \left(n^\beta (\log n)^\gamma X_n\right) \Delta |A_n| n^\epsilon (\log n)^{-\gamma} \sum_{r=1}^{n} r^{-\beta-\epsilon}$$

$$+ O(1) m^\beta X_m |A_m| (\log m)^\gamma m^\epsilon (\log m)^{-\gamma} \sum_{r=1}^{m} r^{-\beta-\epsilon}$$

$$= O(1) \sum_{n=1}^{m} n^{\beta+\epsilon} X_n \Delta |A_n| \left(\int_1^n u^{-\beta-\epsilon} du \right) + O(1) m^{\beta+\epsilon} X_m |A_m| \left(\int_1^m u^{-\beta-\epsilon} du \right)$$

$$= O(1) \sum_{n=1}^{m} n X_n \Delta |A_n| + O(1) m X_m |A_m|$$

$$= O(1) .$$

This establishes (4.2).

5. PROOF OF THE THEOREM:

Let (T_n) be the sequence of $(\overline{N}, p_n)$ mean of the series $\sum_{n=1}^{\infty} a_n \lambda_n$, then

$$T_n = \frac{1}{P_n} \sum_{\upsilon=0}^{n} p_\upsilon \sum_{r=0}^{\upsilon} a_r \lambda_r$$

$$= \frac{1}{P_n} \sum_{\upsilon=0}^{n} (P_n - P_{\upsilon-1}) a_\nu \lambda_\nu$$

Hence for n ≥1

$$T_n - T_{n-1} = \frac{p_n}{P_n P_{n-1}} \sum_{\upsilon=1}^{n} P_{\upsilon-1} a_\upsilon \lambda_\upsilon$$

$$= \frac{p_n}{P_n P_{n-1}} \sum_{\upsilon=1}^{n} \nu a_\nu \left(\frac{1}{\nu} P_{\nu-1} \lambda_\nu \right)$$

$$= \frac{(n+1)}{n} \frac{p_n}{P_n} t_n \lambda_n + \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} p_{\nu-1} t_\nu \lambda_\nu \frac{\nu+1}{\nu} + \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} P_\nu t_\nu \frac{\nu+1}{\nu} \Delta\lambda_\nu$$

$$+ \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} P_\nu t_\nu \frac{\lambda_{\nu+1}}{\nu}$$

$$= T_{n1} + T_{n2} + T_{n3} + T_{n4} \text{(say)}.$$

In order to prove the theorem ,using Minkowski's inequality it is enough to show that

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{\sigma k+k-1} \left| T_{nj} \right| < \infty \quad , j = 1,2,3,4 .$$

Applying Hölder's inequality, we have

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k+k-1} \left| T_{n1} \right|^k = \sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k+k-1} \left| \frac{n+1}{n} \frac{p_n}{P_n} t_n \lambda_n \right|^k$$

$$= O(1) \sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k-1} \frac{|t_n|^k}{X_n^{k-1}} \left(X_n |\lambda_n| \right)^{k-1} |\lambda_n|$$

$$= O(1) \sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k-1} \frac{|t_n|^k}{X_n^{k-1}} |\lambda_n|$$

$$= O(1) \sum_{n=1}^{m-1} \left(\sum_{\upsilon=1}^{n} \left(\frac{P_\nu}{p_\nu} \right)^{\sigma k-1} \frac{|t_\nu|^k}{X_\nu^{k-1}} \right) \Delta |\lambda_n| + O(1) \sum_{\upsilon=1}^{m} \left(\frac{P_\upsilon}{p_\upsilon} \right)^{\sigma k-1} \frac{|t_\nu|^k}{X_\nu^{k-1}} |\lambda_m|$$

$$\le O(1) \sum_{n=1}^{m-1} X_n |A_n| + 0(1) X_m |\lambda_m|$$

$$= O(1) .$$

Next

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k+k-1} \left| T_{n2} \right|^k = \sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k+k-1} \left| \frac{p_n}{P_n P_{n-1}} \sum_{\nu=1}^{n-1} p_{\nu-1} t_\nu \lambda_\nu \frac{\nu+1}{\nu} \right|^k$$

$$= O(1) \sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k-1} \frac{1}{P_{n-1}} \sum_{\nu=1}^{n-1} p_{\nu-1} |t_\nu|^k |\lambda_\nu|^k \left(\sum_{\nu=1}^{n-1} \frac{p_\nu}{P_{n-1}} \right)^{k-1}$$

$$= O(1) \sum_{\nu=1}^{m} p_{\nu-1} |t_\nu|^k |\lambda_\nu|^k \sum_{n=\nu+1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k-1} \frac{1}{P_{n-1}} .$$

$$= O(1) \sum_{\nu=1}^{n} \left(\frac{P_\nu}{p_\nu} \right)^{\sigma k-1} |t_\nu|^k |\lambda_\nu|^k$$

$= O(1)$, as in the case of T_{n1} .

Next

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k+k-1} \left| T_{n3} \right|^k = \sum_{n=1}^{m} \left(\frac{P_n}{p_n} \right)^{\sigma k+k-1} \left| \frac{p_n}{P_n P_{n-1}} \sum_{\upsilon=1}^{n-1} P_\upsilon t_\upsilon \frac{\nu+1}{\nu} \Delta\lambda_\upsilon \right|^k$$

$$= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k-1}\frac{1}{P_{n-1}^k}\sum_{\upsilon=1}^{n-1}P_\nu^k\frac{|t_\upsilon|^k}{X_\nu^{k-1}}|\Delta\lambda_\upsilon|\left(\sum_{\upsilon=1}^{n-1}X_\upsilon|\Delta\lambda_\nu|\right)^{k-1}$$

$$\leq O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k-1}\frac{1}{P_{n-1}^k}\sum_{\upsilon=1}^{n-1}P_\nu^k\frac{|t_\upsilon|^k}{X_\nu^{k-1}}|A_\upsilon|\left(\sum_{\upsilon=1}^{n-1}X_\upsilon|A_\nu|\right)^{k-1}$$

$$= O(1)\sum_{\upsilon=1}^{m}P_\nu^k\frac{|t_\upsilon|^k}{X_\nu^{k-1}}|A_\upsilon|\sum_{n=\upsilon+1}^{m+1}\left(\frac{P_n}{p_n}\right)^{\sigma k-1}\frac{1}{P_{n-1}^k}.$$

$$= O(1)\sum_{\upsilon=1}^{m}\left(\frac{P_\nu}{p_\nu}\right)^{\sigma k}\frac{1}{\nu}\frac{|t_\upsilon|^k}{X_\nu^{k-1}}(\nu|A_\nu|)$$

$$= O(1)\sum_{\upsilon=1}^{m-1}\sum_{r=1}^{\upsilon}\left(\frac{P_r}{p_r}\right)^{\sigma k}\frac{1}{r}\frac{|t_r|^k}{X_r^{k-1}}\Delta(\nu|A_\nu|)+0(m|A_m|)\sum_{r=1}^{m}\left(\frac{P_r}{p_r}\right)^{\sigma k}\frac{1}{r}\frac{|t_r|^k}{X_r^{k-1}}$$

$$= O(1)\sum_{\upsilon=1}^{m-1}\left(-|A_\upsilon|+(\nu+1)\Delta\|A_\upsilon|\right)X_\nu+0(mX_m|A_m|).$$

$$= O(1)\sum_{\upsilon=1}^{n}X_\upsilon|A_\upsilon|+0(1)\sum_{\upsilon=1}^{n}\nu X_\upsilon|\Delta A_\upsilon|+0(mX_m|A_m|)$$

$$\leq O(1)\sum_{\upsilon=1}^{n}X_\upsilon|A_\upsilon|+0(1)\sum_{\upsilon=1}^{n}\nu X_\upsilon\delta_\nu+0(mX_m|A_m|)$$

$$= O(1).$$

Finally

$$\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k+k-1}|T_{n4}|^k = \sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k+k-1}\left|\frac{p_n}{P_nP_{n-1}}\sum_{\nu=1}^{n-1}P_\nu t_\nu\frac{\lambda_{\nu+1}}{\nu}\right|^k$$

$$= O(1)\sum_{n=1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k-1}\frac{1}{P_{n-1}^k}\sum_{\nu=1}^{n-1}\frac{P_\nu}{\nu}|t_\nu|^k|\lambda_\nu|^k\left(\sum_{\nu=1}^{n-1}\frac{P_\nu}{\nu}\right)^{k-1}$$

$$= O(1)\sum_{\nu=1}^{m}\frac{P_\nu}{\nu}|t_\nu|^k|\lambda_\nu|^k\sum_{n=\nu+1}^{m}\left(\frac{P_n}{p_n}\right)^{\sigma k-1}\frac{1}{P_{n-1}}$$

$$= O(1)\sum_{\nu=1}^{m}\left(\frac{P_\nu}{p_\nu}\right)^{\sigma k}\frac{|t_\nu|^k}{X_\nu^{k-1}}(X_\nu|\lambda_\nu|)^{k-1}|\lambda_\nu|$$

$$= O(1)\sum_{\nu=1}^{m}\left(\frac{P_\nu}{p_\nu}\right)^{\sigma k}\frac{|t_\nu|^k|\lambda_\nu|}{\nu X_\nu^{k-1}}$$

$$= O(1)\sum_{\nu=1}^{m-1}\left(\sum_{r=1}^{\nu}\left(\frac{P_r}{p_r}\right)^{\sigma k}\frac{|t_r|^k}{rX_r^{k-1}}\right)|\Delta|\lambda_\nu\|+O(1)\sum_{r=1}^{m}\left(\frac{P_r}{p_r}\right)^{\sigma k}\frac{|t_r|^k}{rX_r^{k-1}}$$

$$\leq O(1)\sum_{\nu=1}^{m-1}\left(\sum_{r=1}^{\nu}\left(\frac{P_r}{p_r}\right)^{\sigma k}\frac{|t_r|^k}{rX_r^{k-1}}\right)|A_\nu|+O(1)\sum_{r=1}^{m}\left(\frac{P_r}{p_r}\right)^{\sigma k}\frac{|t_r|^k}{rX_r^{k-1}}$$

$$= O(1)\sum_{\nu=1}^{m}X_\nu|A_\nu|+O(1)X_m|\lambda_m|$$

$$= O(1).$$

This completes the proof of the theorem.

REFERENCES

[1]. R.P.Boas, " Quasi positive sequences and trigonometric series", Pro. London Math. Soc.. 14A (1965), 38-46

[2]. H.Bor, "A Note on two summability methods", Proc.Amer. Math. Soc.98 (1986), 81-84.

[3]. H.Bor and L.Debnath, "Quasi - β - power increasing sequences", International journal of Mathematics and Mathematical Sciences,44(2004),2371-2376.

[4]. L.Leinder, "A recent note on absolute Riesz summability factors", J. Ineq. Pure and Appl. Math.,Vol-7, Issue-2,article-44(2006).

[5]. U.K.Misra, M.Misra, B.P.Padhy and D.Bisoyi, "On Quasi-f-power increasing sequences", International Mathematical Forum, Vol.8, No5-.8 (2013),377-386

[6]. W.T.Sulaiman, "Extension on absolute summability factors of infinite series", J. Math. Anal. Appl. 322 (2006),1224-1230.

[7]. W.T.Sulaiman, "A recent note on absolute absolute Riesz summability factors of an infinite series", J. Appl. Functional Analysis,Vol-7,no.4,381-387.