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CENTRALIZING AND COMMUTING LEFT GENERALIZED DERIVATIONS ON PRIME RINGS

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Abstract: Let ***R*** be a prime ring and ***d*** a derivation on. ***R*** If ***f*** is a left generalized derivation on ***R*** such that *f* is centralizing on a left ideal ***U*** of ***R*** , then ***R*** is commutative.

Introduction:

H.E Bell and W.S.Martindale III, [1] proved that a semiprime ring ***R*** must have a nontrivial central ideal if it admits an appropriate endomorphism or derivation which is centralizing on ***U***. Then ***R*** is commutative.Bresar.M [2] proved that some concrete additive mappings cannot be centralizing on certain subsets of non-commutative prime rings. He also described the structure of an arbitrary additive mapping which is centralizing on a prime ring. J.H. Mayne [4] had discussed about the existence of a non- trivalautomorphism or derivation which is centralizing on a nonzero ideal in a prime ring and then shown that the ring is commutative. Asif Ali and Tariqshah [5] has considered ***f*** as a generalized derivation on, ***R*** such that ***f*** is centralizing on a left ideal ***U*** of ***R***, then proved that ***R*** is commutative.

Preliminaries:

A ring ***R*** is said to be a prime if ***aRb =0*** implies that either ***a=0*** or ***b=0.*** An additive mapping ***d:R→R*** is said to be derivation if ***d(xy)=d(x)y+xd(y)*** for all ***x,y∈R.*** An additive mapping ***d:R→R*** is said to be left derivation if ***d(xy)=d(x)y+xd(y)*** for ***x,y∈R*** all A mapping ***f*** is said to commuting on a left ideal ***U*** of ***R*** if [***f(x),x***]=***0*** for all ***x∈U*** and ***f*** is said to be centralizing if [***f(x),x***]***∈Z(R)*** for all ***x∈U***. An additive map ***f:R→R*** is called a generalized derivation if there exists a derivation ***R→R*** such that ***f(xy)=f(x)y+xd(y)*** for all ***x,y∈R.*** A additive map ***f:R→R*** is called a left generalized derivation if ***f(xy)=f(x)y+xd(y)*** for all ***x,y∈R*** where ***d*** is derivation on ***R***.
Throughout this paper ***R*** will represent a prime ring with Z(R) being its centre.

Remark1: For a nonzero element $a \in Z(R)$, f $ab \in Z(R)$, then $b \in Z(R)$.
To prove the main results, we find it necessary to establish the following lemma.

Lemma 1: If ***f*** is an additive mapping from **R** to **R** such that ***f*** is centralizing on a left ideal ***U*** of **R**, then for all ***x∈U*** ∩ ***Z(R)*** , ***f(x) ∈Z(R).***

Proof: Since f is centralizing on ***U***,we have
$x, y \in U \Rightarrow x + y \in U$
$\forall\, x + y \in U \Rightarrow [f(x + y), x + y] \in Z(R)$
f is additive then
$[f(x) + f(y), x + y] \in Z(R)$
$\Rightarrow [f(x), x] + [f(x), y] + [f(y), x] + [f(y), y] \in Z(R)$

f is centralizing on left ideal ***U*** then
$[f(x), x] = 0, [f(y), y)] = 0$
$[f(x), y] + [f(y), x] \in Z(R)$ (1)

Now if $x \in Z(R)$, then equation (1) becomes
$[f(x),y]+[f(x),x] \in Z(R)$
$\Longrightarrow [f(x),y] \in Z(R)$ (2)
Replacing ***y*** by ***f(x)y*** in equation (2) we obtain
$[f(x),f(x)y] \in Z(R)$
$\Rightarrow [f(x),f(x)]y+f(x)[f(x),y] \in Z(R)$
$f(x)[f(x),y] \in Z(R)$
Case1: $[f(x),y]=0$
$\forall\ y \in U$ we have $f(x)y-yf(x)=0$
$\Longrightarrow f(x) \in C_R^{(U)}$
The centralizer of ***U*** in ***R*** and hence {[1, identity***IV***]} ***f(x)*** $\in$***Z(R)***..
Case2: $[f(x),y] \neq 0,$
It again follows from remark 1
$x \in Z(R), f(x)[f(x),y] \in Z(R) \Longrightarrow f(x) \in Z(R).$
Hence the lemma.

Theorem1: Let ***R*** be a prime ring. Let ***d:R→R*** be anon zero derivation and ***f*** be a left generalized derivation on a left ideal ***U*** of ***R.*** If is commuting on ***U*** then ***R*** is commutative.

Proof: since ***f*** is commuting on ***U***,we have $[f(x),x]=0\ \forall x \in U.$
Replacing ***x***by ***x+y*** we get
$[f(x+y),x+y]=0$
$[f(x),x]+[f(x),y]+[f(y),x]+[f(y),y]=0$
$\Longrightarrow [f(x),y]+[f(y),x]=0$ (3)
Now by replacing ***y*** by ***xy*** in equation (3) we get
$[f(x),xy]+[f(xy),x]=0$
$[f(x),x]y+x[f(x),y]+[d(x)y+xf(y),x]=0$
$[f(x),x]y+x[f(x),y]+[d(x)y,x]+[xf(y),x]=0$
$[f(x),x]y+x[f(x),y]+[d(x)y,x]+[x,x]f(y)+x[f(y),x]=0$
f is centralizer then, $[f(x),x]y=0, [x,x]f(y)=0.$
$x([f(x),y]+[f(y),x])+[d(x)y,x]=0$ (4)
From (3) we get, $[d(x)y,x]=0.$
Replacing ***y*** by ***yr*** in equation (4) we get
$[d(x)yr,x]=0$
$[d(x)y,x]r+d(x)y[r,x]=0$
From (3) we get
$d(x)y[r,x]=0$
y$\in$***U*** we generalized
$d(x)U[r,x]=0$
r$\in$***RR*** is prime then ***d(x)=0 or [r,x]=0***
$\Longrightarrow \forall x \in U$ either ***x***$\in$***Z(R)*** or **d(x)=0**
Since ***d*** is non-zero on ***R*** then by[4,Lemma 2 $\Longrightarrow d$ is non zero on ***U***
Suppose $d(x) \neq 0$ for some ***x***$\in$***U***
Then ***x***$\in$***Z(R)***
Suppose ***x***$\in$***U*** is such $z \notin Z(R)$ that then ***d(z)=0***
and $x+z \notin Z(R) \Longrightarrow d(x+z)=0 \Longrightarrow d(x)=0$
It is contradiction
$\Longrightarrow z \in Z(R)$ for $z \in U$ for ***z***$\in$***U***
Thus ***U*** is commutative
$\Longrightarrow R$ is commutative
Hence the theorem.

Theorem 2: Let ***U*** be a left ideal of a prime ring ***R*** such that $U \cap z(R) \neq 0$. Let ***d*** be a non zero derivation and ***f*** be a left generalized derivation on ***R*** such that ***f*** is centralizing on ***U***. Then ***R*** is commutative.

Proof: We assume that

Because other wise ***f*** is $Z(R) \neq 0$ ng on ***U*** and there is nothing to prove

Now for anon-zero ***z∈Z(R)***

We replaced ***x*** by ***xy*** in equation (1) we get

$[f(yz),y] + [f(y),yz] \in Z(R)$

$[d(y)z + yf(z),y] + [f(y),yz] \in Z(R)$

$[d(y)z,y] + [yf(z),y] + [f(y),yz] \in Z(R)$

$[d(y),y)]z + d(y)[z,y] + [y,y]f(z) + y[f(z),y] + [f(y),y]z + y[f(y),z] \in Z(R)$

$z \in Z(R) \Longrightarrow [z,y] = 0\ \forall y \in R, [y,y] = 0$

Since $z \in Z(R) \Longrightarrow f$ is centralizer

$f(z) \in Z(R) \Longrightarrow [f(z),y] = 0\ \forall\ y \in R$,nd $[f(y),y] = 0, [f(y),z] = 0$

$\Longrightarrow [d(y),y]z \in Z(R)$

Since ***z*** is non zero it follows that

$[d(y),y] \in Z(R)$

This implies ***d*** is centralizing on ***U*** and hence by [1,Theorem4] we conclude that ***R*** is commutative.

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