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CENTRALIZING AND COMMUTING LEFT GENERALIZED DERIVATIONS ON PRIME RINGS

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Abstract: Let ***R*** be a prime ring and ***d*** a derivation on. ***R*** If ***f*** is a left generalized derivation on ***R*** such that *f* is centralizing on a left ideal ***U*** of ***R*** , then ***R*** is commutative.

Introduction:

H.E Bell and W.S.Martindale III, [1] proved that a semiprime ring ***R*** must have a nontrivial central ideal if it admits an appropriate endomorphism or derivation which is centralizing on ***U***. Then ***R*** is commutative.Bresar.M [2] proved that some concrete additive mappings cannot be centralizing on certain subsets of non-commutative prime rings. He also described the structure of an arbitrary additive mapping which is centralizing on a prime ring. J.H. Mayne [4] had discussed about the existence of a non- trivalautomorphism or derivation which is centralizing on a nonzero ideal in a prime ring and then shown that the ring is commutative. Asif Ali and Tariqshah [5] has considered ***f*** as a generalized derivation on, ***R*** such that ***f*** is centralizing on a left ideal ***U*** of ***R***, then proved that ***R*** is commutative.

Preliminaries:

A ring ***R*** is said to be a prime if $aRb=0$ implies that either $a=0$ or $b=0$. An additive mapping $d:R\rightarrow R$ is said to be derivation if $d(xy)=d(x)y+xd(y)$ for all $x,y\in R$. An additive mapping $d:R\rightarrow R$ is said to be left derivation if $d(xy)=d(x)y+xd(y)$ for $x,y\in R$ all A mapping ***f*** is said to commuting on a left ideal ***U*** of ***R*** if $[f(x),x]=0$ for all $x\in U$ and ***f*** is said to be centralizing if $[f(x),x]\in Z(R)$ for all $x\in U$. An additive map $f:R\rightarrow R$ is called a generalized derivation if there exists a derivation $R\rightarrow R$ such that $f(xy)=f(x)y+xd(y)$ for all $x,y\in R$. A additive map $f:R\rightarrow R$ is called a left generalized derivation if $f(xy)=f(x)y+xd(y)$ for all $x,y\in R$ where ***d*** is derivation on ***R***.
Throughout this paper ***R*** will represent a prime ring with Z(R) being its centre.

Remark1: For a nonzero element $a\in Z(R)$, f $ab\in Z(R)$, then $b\in Z(R)$.
To prove the main results, we find it necessary to establish the following lemma.

Lemma 1: If ***f*** is an additive mapping from **R** to **R** such that ***f*** is centralizing on a left ideal ***U*** of **R**, then for all $x\in U\cap Z(R)$, $f(x)\in Z(R)$.

Proof: Since f is centralizing on ***U***,we have
$x,y\in U\Rightarrow x+y\in U$
$\forall\, x+y\in U\Rightarrow[f(x+y),x+y]\in Z(R)$
f is additive then
$[f(x)+f(y),x+y]\in Z(R)$
$\Rightarrow[f(x),x]+[f(x),y]+[f(y),x]+[f(y),y]\in Z(R)$

f is centralizing on left ideal ***U*** then
$[f(x),x]=0,[f(y),y)]=0$
$$[f(x),y]+[f(y),x]\in Z(R) \quad (1)$$

Now if $x \in Z(R)$, then equation (1) becomes

$[f(x),y] + [f(x),x] \in Z(R)$

$\Rightarrow [f(x),y] \in Z(R)$ (2)

Replacing y by $f(x)y$ in equation (2) we obtain

$[f(x),f(x)y] \in Z(R)$

$\Rightarrow [f(x),f(x)]y + f(x)[f(x),y] \in Z(R)$

$f(x)[f(x),y] \in Z(R)$

Case1: $[f(x),y] = 0$

$\forall\, y \in U$ we have $f(x)y - yf(x) = 0$

$\Rightarrow f(x) \in C_R^{(U)}$

The centralizer of U in R and hence {[1, identityIV]} $f(x) \in Z(R)$..

Case2: $[f(x),y] \neq 0,$

It again follows from remark 1

$x \in Z(R), f(x)[f(x),y] \in Z(R) \Rightarrow f(x) \in Z(R).$

Hence the lemma.

Theorem1: Let R be a prime ring. Let $d{:}R \rightarrow R$ be anon zero derivation and f be a left generalized derivation on a left ideal U of R. If is commuting on U then R is commutative.

Proof: since f is commuting on U,we have $[f(x),x] = 0\ \forall x \in U.$

Replacing x by $x+y$ we get

$[f(x+y),x+y] = 0$

$[f(x),x] + [f(x),y] + [f(y),x] + [f(y),y] = 0$

$\Rightarrow [f(x),y] + [f(y),x] = 0$ (3)

Now by replacing y by xy in equation (3) we get

$[f(x),xy] + [f(xy),x] = 0$

$[f(x),x]y + x[f(x),y] + [d(x)y + xf(y),x] = 0$

$[f(x),x]y + x[f(x),y] + [d(x)y,x] + [xf(y),x] = 0$

$[f(x),x]y + x[f(x),y] + [d(x)y,x] + [x,x]f(y) + x[f(y),x] = 0$

f is centralizer then, $[f(x),x]y = 0, [x,x]f(y) = 0.$

$x([f(x),y] + [f(y),x]) + [d(x)y,x] = 0$ (4)

From (3) we get, $[d(x)y,x] = 0.$

Replacing y by yr in equation (4) we get

$[d(x)yr,x] = 0$

$[d(x)y,x]r + d(x)y[r,x] = 0$

From (3) we get

$d(x)y[r,x] = 0$

$y \in U$ we generalized

$d(x)U[r,x] = 0$

$r \in RR$ is prime then ***d(x)=0 or [r,x]=0***

$\Rightarrow \forall x \in U$ either $x \in Z(R)$ or **d(x)=0**

Since d is non-zero on R then by[4,Lemma 2 $\Rightarrow d$ is non zero on U

Suppose $d(x) \neq 0$ for some $x \in U$

Then $x \in Z(R)$

Suppose $x \in U$ is such $z \notin Z(R)$ that then $d(z)=0$

and $x + z \notin Z(R) \Rightarrow d(x+z) = 0 \Rightarrow d(x) = 0$

It is contradiction

$\Rightarrow z \in Z(R)$ for $z \in U$ for $z \in U$

Thus U is commutative

$\Rightarrow R$ is commutative

Hence the theorem.

Theorem 2: Let ***U*** be a left ideal of a prime ring ***R*** such that $U \cap z(R) \neq 0$. Let ***d*** be a non zero derivation and ***f*** be a left generalized derivation on ***R*** such that ***f*** is centralizing on ***U***. Then ***R*** is commutative.

Proof: We assume that

Because other wise ***f*** is $Z(R) \neq 0$ ng on ***U*** and there is nothing to prove

Now for anon-zero ***z∈Z(R)***

We replaced ***x*** by ***xy*** in equation (1) we get

$[f(yz),y] + [f(y),yz] \in Z(R)$

$[d(y)z + yf(z),y] + [f(y),yz] \in Z(R)$

$[d(y)z,y] + [yf(z),y] + [f(y),yz] \in Z(R)$

$[d(y),y)]z + d(y)[z,y] + [y,y]f(z) + y[f(z),y] + [f(y),y]z + y[f(y),z] \in Z(R)$

$z \in Z(R) \Rightarrow [z,y] = 0\ \forall y \in R, [y,y] = 0$

Since $z \in Z(R) \Rightarrow f$is centralizer

$f(z) \in Z(R) \Rightarrow [f(z),y] = 0\ \forall\ y \in R$,nd $[f(y),y] = 0, [f(y),z] = 0$

$\Rightarrow [d(y),y]z \in Z(R)$

Since ***z*** is non zero it follows that

$[d(y),y] \in Z(R)$

This implies ***d*** is centralizing on ***U*** and hence by [1,Theorem4] we conclude that ***R*** is commutative.

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