A Derivation of Proca Equations on Cantor Sets:
A Local Fractional Approach

Victor Christianto, Biruduganti Rahul

Institute of Gravitation and Cosmology of the Peoples' Friendship University of Russia, Moscow.
victorchristianto@gmail.com

Electronics and Communications, KJ Somaiya College of Engineering, Mumbai.
rahulsid91@gmail.com

Abstract. In a recent paper published at Advances in High Energy Physics (AHEP) journal, Yang Zhao et al. derived Maxwell equations on Cantor sets from the local fractional vector calculus. It can be shown that Maxwell equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. Using the same approach, elsewhere Yang, Baleanu & Tenreiro Machado derived systems of Navier-Stokes equations on Cantor sets. However, so far there is no derivation of Proca equations on Cantor sets. Therefore, in this paper we present for the first time a derivation of Proca equations and GravitoElectroMagnetic (GEM) Proca-type equations on Cantor sets. Considering that Proca equations may be used to explain electromagnetic effects in superconductor, We suggest that Proca equations on Cantor sets can describe electromagnetic of fractal superconductors; besides GEM Proca-type equations on Cantor sets may be used to explain some gravitoelectromagnetic effects of superconductor for fractal media. It is hoped that this paper may stimulate further investigations and experiments in particular for fractal superconductor. It may be expected to have some impact to fractal cosmology modeling too.

1. Introduction

According to the late Benoit Mandelbrot, fractal geometry is a workable geometric middle ground between excessive geometric order of Euclid and the geometric chaos of general mathematics. It is based on a form of symmetry that had previously been underused, namely invariance, under contraction or dilation. [1] Fractal geometry has many applications including in biology, physics, geophysics, engineering, mathematics, cosmology and other fields of science and art. A rapidly growing field is to express electromagnetic wave equations in fractal media.

The present paper is intended to be a follow-up paper of my three recent papers: one paper reviews Shpenkov's interpretation of classical wave equation and its role to explain periodic table of elements and other phenomena [16], and the second one presents a derivation of GravitoElectroMagnetic Proca equations in fractional space [19], and the third one presents an outline of cosmology based on the concept of fractal vibrating string [28].

The idea for writing the present paper comes from George Shpenkov's papers, where he shows that correct interpretation of classical wave equation yields a periodic table of elements which is close to Mendeleyev's periodic law.[13][14][15] From that result he is able to derive many results corresponding to the structure of neutron, proton, and molecules based on classical wave equation:

$$\Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0$$

$$(1)$$

This equation is also known as the wave equation of sound or string vibration. George Shpenkov's work is based on: (1) Dialectical philosophy and dialectical logic; (2) The postulate on the wave nature of all phenomena and objects in the Universe.[12]

Now the question is: Is it possible to hypothesize that the entire Universe consists of sound wave and vibration and frequency, just like atoms and molecules? Interestingly, Leonardo Rubino puts forth hat conjecture based on the same classical wave equation. [20][21] He hypothesizes that the frequency of the Universe is: [22]

$$f_{Univ} = 4,047 \cdot 10^{-21} \, Hz$$

(2)

One persistent question in this regard is: How to explain photon as quanta and also photoelectric effect from this wave picture? Interestingly, Xin-an Zhang has provided an outline of answer to that question, which will be described as follows.[23] In his approach, the electromagnetic force is regarded as deferring to the sine function, reach the highest at the position of 1/4 wavelength. At his point, if the highest force is not able to move the particle, the particle will never been moved because the succeeding force will drop down with the law of sine function. That means, the energy transmission will occur only in the front of 1/4 wavelength of the light. As shown in Figure 1, the force f that the light wave strikes on the electron is $f = F \sin \varphi$, where F is the maximal value of force, φ is the phase angle.
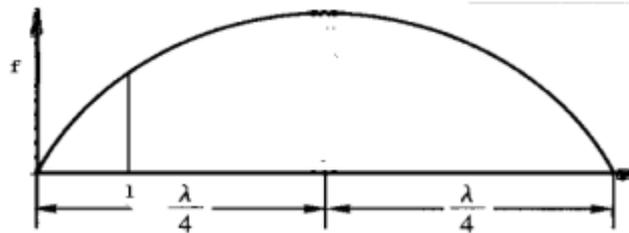

Figure 1. The force deferring to sine function acting on the particle [23]

When the displacement on the abscissa is l, the phase angle will be $2\pi \frac{l}{\lambda}$ and $f = F \sin(2\pi \frac{l}{\lambda})$.

Given that s is the displacement of the particle been pushed by the light wave and S is its maximal value, then the work that the wave force to the particle will be

$$W = \int_0^S F \sin(2\pi \frac{l}{\lambda}) ds,$$

(3)

where the sine function can be expanded

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

(4)

In accordance with the above discussion, the energy transmission merely happens at the front 1/4 wavelength. Thus, the $x = 2\pi \frac{l}{\lambda}$ is smaller than 1. And we get $\sin(2\pi \frac{l}{\lambda}) \approx \frac{2\pi l}{\lambda}$, then we substitute it into Eq. (3), finishing the integral and considering the energy has been transmitted totally, then we get

$$E = W = F \sin(2\pi \frac{l}{\lambda}) S = 2\pi F S \frac{l}{\lambda}$$

(5)

Designating c, λ, v and the light speed, wavelength and frequency separately, considering $l = ct$ and setting

$$h = 2\pi F S \frac{l}{c} = 2\pi F t S$$

(6)

Hence, we get

$$E = hv \tag{7}$$

It can be concluded therefore, that the quanta of photon can be described from a wave viewpoint too. Xin-an Zhang is also able to explain Compton effect, atomic hydrogen spectrum formula, as well as the blackbody radiation from the viewpoint of wave vibration [23].

Therefore it appears interesting to generalize further the wave equation of sound, in particular considering new results in fractal geometry studies, as follows:

    a. To generalize the wave equation of sound (1) to become fractal vibrating string or fractal wave equation;

    b. To generalize the wave equation to become Maxwell equations and Proca equations for massive photon. Such a generalization is possible because when the non-differentiable terms are removed from Maxwell equations, it can easily be shown that the components of electrical field-strength and the components of magnetic field-strengths all satisfy the standard wave equation:

$$\nabla^2 \phi = \left(\frac{1}{\overline{c}^2} \right) \frac{\partial^2 \phi}{\partial t^2} \text{, see Thornhill [25].}$$

    c. To generalize further Maxwell equations and Proca equations on Cantor sets.

For point a), it has been suggested in   a recent paper to write down the wave equation on Cantor sets (local fractional wave equation) as follows: [24, p.2]

$$\frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} - a^{2\alpha} \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} = 0$$

$$\tag{8}$$

where the operators are local fractional ones. For other approaches, see [17][18].

In this regard, in a recent paper Yang Zhao et al. derived Maxwell equations on Cantor sets from the local fractional vector calculus.[2] It can be shown that Maxwell equations on Cantor sets in a ractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. Using the same approach, elsewhere Yang, Baleanu & Tenreiro Machado derived systems of Navier-Stokes equations on Cantor sets.[11] However, so far there is no derivation of Proca equations and GravitoElectroMagnetic Proca-type equations on Cantor sets. Therefore, in this paper I present for the first time a derivation of Proca equations and GravitoElectroMagnetic (GEM) Proca-type equations on Cantor sets. Considering that Proca equations may be used to explain electromagnetic effects in superconductor, I suggest that GEM Proca-type equations on Cantor sets may be used to explain some gravitoelectromagnetic effects of superconductor for fractal media. It is hoped that this paper may stimulate further investigations and experiments on gravitomagnetic effects in particular for superconductor. It may be expected to have some impact to fractal cosmology modeling too.

It shall be noted that the present paper is not intended to be a complete description of fractal gravitatoelectromagnetic wave theory on Cantor sets. Instead, this paper is intended to stimulate further investigations and experiments related to gravitoelectromagnetic effects of superconductors in fractal media and their implications to fractal cosmology modeling.

2. A review of previous result - Maxwell equations on Cantor sets

I will not re-derive Maxwell equations here. For a good reference on Maxwell equations, see for example Julian Schwinger et al.'s book: Classical Electrodynamics [9]. Penrose also discusses Maxwell equations shortly in his book: The Road to Reality[10]. Zhao et al. were able to write the local fractional differential forms of Maxwell equations on Cantor sets as follows [2, p.4-5]:

- Gauss's law for the fractal electric field: $\nabla^\alpha \cdot D = \rho$,   (9)

- Ampere's law in the fractal magnetic field: $\nabla^\alpha \times H = J_a + \dfrac{\partial^\alpha D}{\partial t^\alpha}$,   (10)

- Faraday's law in the fractal electric field: $\nabla^\alpha \times E = -\dfrac{\partial^\alpha B}{\partial t^\alpha}$,   (11)

- magnetic Gauss's law in the fractal magnetic field: $\nabla^\alpha \cdot B = 0$,   (12)

and the continuity equation can be defined as:

$$\nabla^\alpha \cdot J = -\frac{\partial^\alpha \rho}{\partial t^\alpha},$$ (13)

where $\nabla^\alpha \cdot r$ and $\nabla^\alpha \times r$ are defined as follows:

2.1. In Cantor coordinates [11, p. 2]:

$$\nabla^\alpha \cdot u = div^\alpha u = \frac{\partial^\alpha u_1}{\partial x_1^\alpha} + \frac{\partial^\alpha u_2}{\partial x_2^\alpha} + \frac{\partial^\alpha u_3}{\partial x_3^\alpha},$$ (14)

$$\nabla^\alpha \times u = curl^\alpha u = \left(\frac{\partial^\alpha u_3}{\partial x_2^\alpha} - \frac{\partial^\alpha u_2}{\partial x_3^\alpha} \right) e_1^\alpha + \left(\frac{\partial^\alpha u_1}{\partial x_3^\alpha} - \frac{\partial^\alpha u_3}{\partial x_1^\alpha} \right) e_2^\alpha + \left(\frac{\partial^\alpha u_2}{\partial x_1^\alpha} - \frac{\partial^\alpha u_1}{\partial x_2^\alpha} \right) e_3^\alpha.$$ (15)

2.2. In Cantor-type cylindrical coordinates [2, p.4]:

$$\nabla^\alpha \cdot r = \frac{\partial^\alpha r_R}{\partial R^\alpha} + \frac{1}{R^\alpha} \frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} + \frac{r_R}{R^\alpha} + \frac{\partial^\alpha r_z}{\partial z^\alpha},$$ (16)

$$\nabla^\alpha \times r = \left(\frac{1}{R^\alpha} \frac{\partial^\alpha r_\theta}{\partial \theta^\alpha} - \frac{\partial^\alpha r_\theta}{\partial z^\alpha} \right) e_R^\alpha + \left(\frac{\partial^\alpha r_R}{\partial z^\alpha} - \frac{\partial^\alpha r_z}{\partial R^\alpha} \right) e_\theta^\alpha + \left(\frac{\partial^\alpha r_\theta}{\partial R^\alpha} + \frac{r_R}{R^\alpha} - \frac{1}{R^\alpha} \frac{\partial^\alpha r_R}{\partial \theta^\alpha} \right) e_z^\alpha.$$ (17)

It is worth noting here, that Martin Ostoja-Starzewski has derived Maxwell equations in anisotropic fractal media using a different method. [3]

3. Proca Equations on Cantor Sets

Proca equations can be considered as an extension of Maxwell equations, and they have been derived in various ways, see for instance [4, 6, 7]. It can be shown that Proca equations can be derived from first principles [6], and also that Proca equations may have link with Klein-Gordon equation [7]. However, in this paper I will not attempt to re-derive Proca equations. Instead, I will use Proca equations as described in [6]. Then I will derive the Proca equations on Cantor Sets, in accordance with Zhao et al.'s approach as outlined above [2].

According to Blackledge, Proca equations can be written as follows [7]

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2 \phi,$$ (18)

$$\nabla \cdot \vec{B} = 0,$$ (19)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},$$ (20)

$$\nabla \times \vec{B} = \mu_0 j + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \kappa^2 \vec{A},$$ (21)

where:

$$\nabla\phi = -\frac{\partial\vec{A}}{\partial t} - \vec{E}, \tag{22}$$

$$\vec{B} = \nabla\times\vec{A}, \tag{23}$$

$$\kappa = \frac{mc_0}{\hbar}. \tag{24}$$

Therefore, by using the definitions in equations (14)-(17), we can arrive at Proca equations on Cantor sets from (18) through (23), as follows:

$$\nabla^\alpha\cdot\vec{E} = \frac{\rho}{\varepsilon_0} - \kappa^2\phi, \tag{25}$$

$$\nabla^\alpha\cdot\vec{B} = 0, \tag{26}$$

$$\nabla^\alpha\times\vec{E} = -\frac{\partial^\alpha\vec{B}}{\partial t^\alpha}, \tag{27}$$

$$\nabla^\alpha\times\vec{B} = \mu_0 j_\alpha + \varepsilon_0\mu_0\frac{\partial^\alpha\vec{E}}{\partial t^\alpha} + \kappa^2\vec{A}, \tag{28}$$

where:

$$\nabla^\alpha\phi = -\frac{\partial^\alpha\vec{A}}{\partial t^\alpha} - \vec{E}, \tag{29}$$

$$\vec{B} = \nabla^\alpha\times\vec{A}, \tag{30}$$

and Del operator $\nabla^\alpha\phi$ can be defined as follows [11, p.2]:

$$\nabla^\alpha\phi = \frac{\partial^\alpha\phi}{\partial x_1^\alpha}e_1^\alpha + \frac{\partial^\alpha\phi}{\partial x_2^\alpha}e_2^\alpha + \frac{\partial^\alpha\phi}{\partial x_3^\alpha}e_3^\alpha. \tag{31}$$

To my best knowledge so far, the above expressions of Proca equations on Cantor sets (25)-(30) have not been proposed elsewhere before.

Since according to Blackledge, the Proca equations can be viewed as a unified wavefield model of electromagnetic phenomena [7], therefore we can also regard the Proca equations on Cantor sets as a further generalization of Blackledge's unified wavefield model.

It appears interesting to remark here, that Luke Kenneth Casson Leighton [26] recently introduces an expansion of the Rishon Model to cover quark generations. He only uses a simple assumption that all particles in effect photons phase-locked in a repeating pattern inherently obeying Maxwell equations. Therefore, it may be expected that Proca equations on Cantor sets may have some impacts on the nature of Rishon Model.

One persistent question concerning these Proca equations is how to measure the mass of the photon. This question has been discussed in lengthy by Tu, Luo & Gillies [27]. According to their report, there are various methods to estimate the upper bound limits of photon mass. In Table 1 below, some of upper bound limits of photon mass based on dispersion of speed of light are summarized.

Table 1. Upper bound on the dispersion of the speed of light in different ranges of the electromagnetic spectrum, and the corresponding limits on the photon mass. [27, p.94]

Author (year)	Type of measurement	Limits on m_γ (g)
Ross et al. (1937)	Radio waves transmission overland	5.9×10^{-42}
Mandelstam & Papalexi (1944)	Radio waves transmission over sea	5.0×10^{-43}
Al'pert et al. (1941)	Radio waves transmission over sea	2.5×10^{-43}
Florman (1955)	Radio-wave interferometer	5.7×10^{-42}
Lovell et al. (1964)	Pulsar observations on flour flare stars	1.6×10^{-42}
Frome (1958)	Radio-wave interferometer	4.3×10^{-40}
Warner et al. (1969)	Observations on Crab Nebula pulsar	5.2×10^{-41}
Brown et al. (1973)	Short pulses radiation	1.4×10^{-33}
Bay et al. (1972)	Pulsar emission	3.0×10^{-46}
Schaefer (1999)	Gamma ray bursts	4.2×10^{-44}
	Gamma ray bursts	6.1×10^{-39}

From this table and also from other results as reported in [27], it seems that we can expect that someday photon mass can be observed within experimental bound.

4. GravitoElectroMagnetic (GEM) Proca-type Equations on Cantor Sets

The term GravitoElectroMagnetism (GEM) refers to the formal analogies between Newton's law of gravitation and Coulomb's law of electricity. The theoretical analogy between the electromagnetic and the gravitational field equations has been first suggested by Heaviside in 1893, see for example [8]. The fields of GEM can be defined in close analogy with the classical electrodynamics. Therefore, if we can consider Proca equations as generalization and extension of Maxwell equations, then we can also find **GravitoElectroMagnetic** Proca-type equations.

In accordance with Demir [8], the GravitoElectroMagnetic Proca-type equations can be expressed straightforward from their electromagnetic counterpart as follows (Here I use Demir's notations instead of Blackledge's notations):

$$\nabla \cdot \vec{E}_g = -\rho_e - \kappa_g^2 \phi, \tag{32}$$

$$\nabla \cdot \vec{H}_g = 0, \tag{33}$$

$$\nabla \times \vec{E}_g = -\frac{\partial \vec{H}_g}{\partial t}, \tag{34}$$

$$\nabla \times \vec{H}_g = -J_g^e + \frac{\partial \vec{E}_g}{\partial t} + \kappa_g^2 \vec{A}_g^e, \tag{35}$$

where the fields E_g and H_g can be defined in terms of the potentials just as given in equation (22) and (23), and the term k_g represents the inverse Compton wavelength of the graviton, [8]

$$\kappa_g = \frac{m_g c}{\hbar}. \tag{36}$$

$$\nabla^\alpha \cdot \vec{E}_g = -\rho_e - \kappa_g^2 \phi, \tag{37}$$

$$\nabla^\alpha \cdot \vec{H}_g = 0, \tag{38}$$

$$\nabla^\alpha \times \vec{E}_g = -\frac{\partial^\alpha \vec{H}_g}{\partial t^\alpha}, \tag{39}$$

$$\nabla^\alpha \times \vec{H}_g = -J_g^e + \frac{\partial^\alpha \vec{E}_g}{\partial t^\alpha} + \kappa_g^2 \vec{A}_g^e, \tag{40}$$

To my best knowledge so far, the above expressions of GravitoElectroMagnetic Proca equations on Cantor sets (37)-(40) have not been proposed elsewhere before. It will be interesting to conduct experiments to measure on how extent these equations on Cantor sets differ from the GEM Proca equations [4][5].

Concluding remarks

In a recent paper Yang Zhao et al. derived Maxwell's equation on Cantor sets from the local fractional vector calculus. It can be shown that Maxwell's equations on Cantor sets in a fractal bounded domain give efficiency and accuracy for describing the fractal electric and magnetic fields. Using the same approach, elsewhere Yang, Baleanu & Tenreiro Machado derived systems of Navier-Stokes equations on Cantor sets. However, so far there is no derivation of Proca equations and GravitoElectroMagnetic Proca-type equations on Cantor sets. Therefore, in this paper I present for the first time a derivation of Proca equations and GravitoElectroMagnetic (GEM) Proca-type equations on Cantor sets. Considering that Proca equations may be used to explain electromagnetic effects in superconductor, I suggest that GEM Proca-type equations on Cantor sets may be used to explain some gravitoelectromagnetic effects of superconductor for fractal media. It is hoped that this paper may stimulate further investigations and experiments on gravitomagnetic effects in particular for superconductor. It may be expected to have some impact to fractal cosmology modeling too.

It shall be noted that the present paper is not intended to be a complete description of fractal gravitation wave theory on Cantor sets. Instead, this paper is intended to stimulate further investigations and experiments related to gravitomagnetic effect of superconductors and their implications to fractal cosmology modeling. This kind of investigation may be useful for the study of gravitomagnetic effects.

Of course, any generalization and simplification have its own risk, but we should also remember that Schrödinger himself considered that everything is wave, although he failed to convince anyone else.

Furthermore, there is a wave function model of universe known as Wheeler-DeWitt equation, which is quite popular in quantum cosmology study. However, it is known that WDW equation lacks observational support. Therefore we hope that using the wave equation we may obtain better results.

By suggesting that Universe can be modeled as a wave, we wish to push the boundary of observation limit. Only time will tell if this endeavor will yield something.

Acknowledgments:

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