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ON $\hat{\alpha}$g COMPACT SPACES

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ABSTRACT. We introduce and study some subsets of a topological space X called $\hat{a}$ g compact sets; $\hat{a}$ g compact spaces are defined and their properties are studied.

1. INTRODUCTION

Throughout the paper (X,τ) denotes a topological space in which no separation axiom is assumed.(X,τ) will be simply denoted by X. Levine [3] initiated semi open sets in topological spaces. Several spaces are defined in terms of semi open sets such as S – Compact [5], S – closed space [10], s-closed space [2] etc. In section 3 of the present work, we introduce the notion of α g compact spaces and study their properties.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space X is said to be
1.pre open [4] if A ⊂int cl A and pre closed, if cl int A⊂ A.
2.regular open [4] if A= int cl A and regular closed if A = cl int A.
3.semi open [4] if A ⊂cl int A and semi closed if int cl A⊂A.
Definition 2.2: A subset A of a topological space X is said to $\hat{a}$ g closed if int cl int A⊂ U whenever A⊂U and U is open in X.
The complement of $\hat{a}$ g closed set in X is called $\hat{a}$ g open set in X.
The union of all $\hat{a}$ g open sets contained in A is called $\hat{a}$ g interior of A and is denoted by $\hat{a}$ g int A. In general $\hat{a}$ g int A is not $\hat{a}$ g open.
The intersection of all $\hat{a}$ g closed sets containing A is called the $\hat{a}$ g closure of A and is denoted by $\hat{a}$ g cl A. In general $\hat{a}$ g cl A is not $\hat{a}$ g closed.
In what follows we assume the intersection of $\hat{a}$ g closed sets is $\hat{a}$ g closed. Then $\hat{a}$ gint A is $\hat{a}$ g open and $\hat{a}$ g cl A is $\hat{a}$ g closed. Also we assume the union of $\hat{a}$ g closed sets is $\hat{a}$ g closed.

3. $\hat{\alpha}$g COMPACT SPACES

Definition 3.1: By a $\hat{a}$ g open cover (or covering) of a subset A of a topological space X, we mean a collection **C** = { Gλ: λ∈∇ } of $\hat{a}$ g open subsets of X such that A⊂ ∪{ Gλ: λ∈∇ }
Definition 3.2: A subset A of a topological space X is said to be $\hat{a}$ g compact if and only if every $\hat{a}$ g open cover of A has a finite subcover.
In particular, X is said to be $\hat{a}$ g compact if and only if for every collection { Gλ: λ∈∇ } of $\hat{a}$ g open sets for which X=∪{ Gλ: λ∈∇ } there exists finitely many sets Gλ1,Gλ2,………Gλn among the Gλ such that X = ∪{Gλ i: i = 1,2,…n}

Remark 3.3: Finite union of $\hat{a}$ g compact sets is compact.
Definition 3.4: A topological space is said to be $\hat{a}$ g Hausdorff if for distinct points x,y of X, there exist disjoint $\hat{a}$ g open sets M and N containing x and y respectively.
Theorem 3.5: Every $\hat{a}$ g compact subset of a $\hat{a}$ g Hausdorff space is closed.
Proof: Let A be a $\hat{a}$ g compact subset of a $\hat{a}$ g Hausdorff space X.
Let us prove A' is $\hat{a}$ g open. Let p∈A'.
As X is $\hat{a}$ g Hausdorff , for every q∈A, there exist disjoint α g open sets M (q) and N(q) of p and q respectively.
The collection { N (q): q∈A} is a $\hat{a}$ g open cover of A. Since A is $\hat{a}$ g compact, there exist finite number of qi, s,i= 1,2,……n such that
A⊂ ∪{N qi) : i= 1,2,…n}
Let M =∩ {M(qi) : i = 1,2,….n}
N=∪{N(qi) : i = 1,2,….n}
M is a $\hat{a}$ g open set containing p. We have x∈N ⇒ x∈N(qi), for some i
Therefore x∉M(qi) and hence x ∉ M
Hence M∩ N = ϕ. A⊂ N. So, M∩ A = ϕ
So, M⊂A'. A' contains $\hat{a}$ g neighbourhood of each of its points. So, A' is $\hat{a}$ g open and hence A is $\hat{a}$ g closed.
Theorem 3.6: Every $\hat{a}$ g closed subset of a compact space is $\hat{a}$ g compact.
Proof: Let F be a $\hat{a}$ g closed subset of a $\hat{a}$ g compact space X. Let **C** = { Gλ : λ∈∇} be a $\hat{a}$ g open cover of F. Then the collection **D**= { Gλ: λ ∈∇ }∪ {X – F} forms $\hat{a}$ g open cover of X. Since X is compact, there is a finite subcollection **D'** of **D** that covers X.
If X-F is a member of **D'**, remove it from **D'**. The remaining finite collection will cover F.So F is $\hat{a}$ g compact.
Theorem 3.7: A topological space X is $\hat{a}$ g compact if and only if every collection of $\hat{a}$ g closed subsets of X with finite intersection property (FIP) has a nonempty intersection.
Proof: Let X be $\hat{a}$ g compact. Let F = { Fλ : λ ∈∇ } be a collection of $\hat{a}$ g closed subsets of X with the FIP .Suppose, ∩{Fλ: λ ∈∇} =ϕ. Then∪{F'λ: λ ∈∇} =X,by De Morgan's law.
This means the collection {F'λ: λ ∈∇}is a $\hat{a}$ g open cover of X. Since X is compact, we have X=∪{F'λi : i = 1,2,…n}, where n is finite.
Again by De morgan's law(∩{Fλi : i = 1,2,…,n})' = X
This implies ∩{ Fλi: i= 1,…n} = ϕ contradicting FIP. Hence ∩{Fλ:λ∈∇} ≠ϕ
Conversely, let every collection of $\hat{a}$ g closed subsets of X with FIP have nonempty intersection.
Let **C** = { Gλ : λ ∈∇}be a $\hat{a}$ g open cover of X. X =∪{ Gλ : λ ∈∇}. ϕ=∩{ G'λ : λ ∈∇}.
Thus { G'λ : λ ∈∇} is a collection of $\hat{a}$ g closed sets with empty intersection. So, this collection does not have the FIP. Hence there exist a finite number of sets Gλi , i = 1,2,…,n such that ϕ=∩{G'λi : i=1,2,…,n} = (∪ { Gλi :i=1,2,…,n})'
This implies X = ∪ { Gλi :i=1,2,…,n}).
Hence X is $\hat{a}$ g compact.
Theorem 3.8: A topological space X is $\hat{a}$ g compact if and only if every class of $\hat{a}$ g closed sets with empty intersection has a finite sub class with empty intersection.
Proof: Let X be a $\hat{a}$ g compact space. Let **C**= {Fα :α∈∇}be a collection of $\hat{a}$ g closed subsets of X such that∩ {Fα :α∈∇ } = ϕ
Taking complements ∪{F'α :α∈∇ } = X
The collection {F'α :α∈∇ } is a $\hat{a}$ g open covering of X. As X is $\hat{a}$ g compact, there exist finite number of indices α1, α2,….αn such that
∪ {F'αi:i=1,2,…n } = X
Taking complements ∩{Fαi :i=1,2,…n } = ϕ
Conversely, suppose that every class of $\hat{a}$ g closed subsets of X with empty intersection has a finite subclass with empty intersection. Let**C***={Gα : α∈∇} be an arbitrary $\hat{a}$ g open covering of X.

$\cup\{G\alpha : \alpha\in\nabla\} = X$.
Taking complements,$\cap\{G'\alpha : \alpha\in\nabla\} = .\ \phi$
The collection $\{G'\alpha : \alpha\in\nabla\}$ has a empty intersection. Hence, there exist finite number of indices α1, α2,….αn such that,
$\cap\{G'\alpha i : i = 1,2,\ldots,n\} = \phi$. Taking complements. $\cup\{G\alpha i : i = 1,2,\ldots,n\} = X$.
Hence X is $\hat{a}$ g compact.

Definition 3.9: A topological space X is said to have α g Bolzano – Weierstrass property ($\hat{a}$ g BWP) if and only if every infinite set in X has a $\hat{a}$ g limit point.

Theorem 3.10: If A is an infinite subset of a $\hat{a}$ g compact space X, then A has a $\hat{a}$ g limit point.

Proof: Let A have no $\hat{a}$ g limit point in X. Then, for every $x\in X$, there exists a $\hat{a}$ g neighbourhood Nx of x which contains no point of A other than (possibly) x. Now the collection $\{Nx: x \in X\}$ forms a $\hat{a}$ g open cover of X. Since, X is $\hat{a}$ g compact, there exist finitely many points x1,x2,,…..xn in X such that $X = \cup\{Nxi : i= 1,2,\ldots n\}$
Consequently $A \subset\cup\{Nxi : i= 1,2,\ldots n\}$. As each Nx contains atmost one point of A, the above relation shows A has atmost n points and hence A is finite, which is a contradiction.
This completes the proof.

Remark 3.11 : No infinite discrete space is $\hat{a}$ g compact.

Theorem 3.12 : Let A and B be disjoint $\hat{a}$ g compact subsets of a $\hat{a}$ g Hausdorff space X. Then there exist disjoint $\hat{a}$ g open subsets G and H such that $A\subset G$, $B\subset H$.

Proof: Let $a\in A$ be arbitrary but fixed. Let $x\in B$ be arbitrary. As $A\cap B = \phi$, $a\neq x$.
There exist disjoint $\hat{a}$ g open subsets Gx and Hx such that $a\in Gx$, $x\in Hx$. The collection $\{Hx : x\in B\}$ is a $\hat{a}$ g open cover of B. As B is $\hat{a}$ g compact, there exist finitely many points $x1,x2,\ldots xn\in B$ such that $B\subset\cup\{Hxi : i= 1,2,\ldots n\}=Ha$ (say)
Let $Ga = \cap\{Gxi: i=1,2,\ldots n\}$. Then Ga and Ha are disjoint $\hat{a}$ g open sets such that $a\in Ga$, $B\subset Ha$..
$\{Ga : a\in A\}$ is a $\hat{a}$ g open cover of A. As A is $\hat{a}$ g compact, there exist finitely many points a1,a2, ….. $am\in A$ such that
$A \subset\cup \{Gai : i= 1,2, \ldots m\}=G$ (say).
Let $H = \cap\{Hai ; i = 1,2,\ldots m\}$. Then G and H are disjoint $\hat{a}$ g open sets such that $A\subset G$ and $B\subset H$.

Theorem 3.13 ; Let X be a $\hat{a}$ g connected $\hat{a}$ g Hausdorff space[8]. Then no nonempty proper $\hat{a}$ g open subset of X is $\hat{a}$ g compact.

Proof :X is $\hat{a}$ g connected. No nonempty $\hat{a}$ g open proper subset of X can be $\hat{a}$ g closed. Thus, if G is a nonempty proper $\hat{a}$ g open subset of X, it is not $\hat{a}$ g closed. A $\hat{a}$ g compact subset of a $\hat{a}$ g Hausdroff space is closed. Since G is not $\hat{a}$ g closed, it is not $\hat{a}$ g compact.

Theorem 3.14 : If the product of two nonempty spaces is $\hat{a}$ g compact, then each factor space is $\hat{a}$ g compact.

Proof : Let XxY be the product space of the nonempty spaces X and Y and let XxY be $\hat{a}$ g compact.
Then the projection $\pi1:XxY\rightarrow X$ is a $\hat{a}$ g irresolute map. Hence $\pi1$ (XxY) = X is α g compact.
For the space Y, the proof is similar.

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