SEPARATION AXIOMS IN QUAD TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce separation axioms in quad topological space (q topological spaces) and study some of their properties.

1. INTRODUCTION


2. PRILIMINARIES

Definition 2.1 [4]: Let \( X \) be a nonempty set and \( \tau_1, \tau_2, \tau_3 \) and \( \tau_4 \) are general topologies on \( X \). Then a subset \( A \) of space \( X \) is said to be quad-open(q-open) set if \( A \subseteq \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4 \) and its complement is said to be q-closed and set \( X \) with four topologies called \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \). q-open sets satisfy all the axioms of topology.

Definition 2.2 [4]: A subset of a q-topological space \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \) is called q-neighborhood of a point \( x \in X \) if and only if there exist q-open sets such that \( x \subseteq X \subseteq A \).

Note 2.3[4]: We will denote the q-interior (resp. q-closure) of any subset, say of by q-int\( A \) (q-cl\( A \)), where q-int\( A \) is the union of all q-open sets contained in \( A \), and q-cl\( A \) is the intersection of all q-closed sets containing \( A \).

3. SEPARATION AXIOMS IN QUAD TOPOLOGICAL SPACES

Definition 3.1: A quad (q) topological space \( X \) is said to be \( q-T_0 \) space iff to given any pair of distinct points \( x,y \) in \( X \), there exists a q-open set containing one of the points but not the other.

Example 3.2: Let \( X = \{a, b, c\} \), \( \tau_1 = \{X, \emptyset, \{a\}\} \), \( \tau_2 = \{X, \emptyset, \{a\}, \{a, b\}\} \), \( \tau_3 = \{X, \emptyset, \{a, b\}\} \), \( \tau_4 = \{X, \emptyset\} \), all q-open sets are \( X, \emptyset, \{a\}, \{a, b\} \) so \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \) is q-space.

Theorem 3.3: If \( \{x\} \) is q-open for some \( x \in X \) then \( x \notin q-cI\{y\} \), for all \( y \neq x \).
Proof: Let \( \{x\} \) be a q-open for some \( x \in X \), then \( X - \{x\} \) is q-closed and \( x \notin X - \{x\} \). If \( x \in q - cl(y) \), for some \( y \neq x \), then \( y, x \) both are in all the q-closed sets containing \( y \), so \( x \in X - \{x\} \) which is contradiction, hence \( x \notin q - cl(y) \).

Theorem 3.4: In any q-topological space \( X \), any distinct points have distinct q-closures.

Proof: Let \( x, y \in X \) with \( x \neq y \), & let \( A = \{x\} \) hence \( q - cl(A) = A \) or \( X \). Now if \( cl(A) = A \) then \( A \) is q-closed so \( X - A = \{x\} \) is q-open & not containing \( y \). So by theorem(3.3) \( x \notin q - cl(\{y\}) \) & \( y \in q - cl(\{y\}) \) which implies that \( q - cl(\{y\}) \) & \( q - cl(\{x\}) \) are distinct. If \( q - cl(A) = X \) then \( A \) is q-open, hence \( \{x\} \) is q-closed, which mean that \( q - cl(\{x\}) = \{x\} \) which is not equal to \( q - cl(\{y\}) \).

Theorem 3.5: In any quad topological space \( X \), if distinct points have distinct q-\( T_0 \) closures then \( X \) is q- \( T_0 \) space.

Proof: Let \( x, y \in X \) with \( x \neq y \), with \( q - cl(\{y\}) \) is not equal to \( q - cl(\{x\}) \), hence there exists \( z \in X \) such that \( z \in q - cl(\{x\}) \), but \( z \notin q - cl(\{y\}) \) or \( z \in q - cl(\{y\}) \) but \( z \notin q - cl(\{x\}) \).

Now, without loss of generality, let \( z \in q - cl(\{x\}) \), but \( z \in q - cl(\{y\}) \). If \( x \in q - cl(\{y\}) \), then \( q - cl(\{x\}) \) is contained in \( q - cl(\{y\}) \) hence \( z \notin q - cl(\{y\}) \) which is a contradiction, this mean that \( x \notin q - cl(\{y\}) \) hence \( x \in q - cl(\{y\}) \), hence \( X \) is q- \( T_0 \) space.

Definition 3.6: A quad –topological space \( X \) is said to be q- \( T_1 \) space iff to given any pair of distinct point \( x \) & \( y \) of \( X \) there exist two q-open sets \( U, V \) such that \( x \in U, y \notin U \) & \( y \in V, x \notin V \).

Example 3.7: Let \( X = \{a, b, c\} \), \( \tau_1 = \{X, \emptyset, \{a\}, \{a, b\}\} \), \( \tau_2 = \{X, \emptyset, \{a\}\} \), \( \tau_3 = \{X, \emptyset, \{a, c\}\} \), \( \tau_4 = \{X, \emptyset, \{b\}\} \) all q-open sets are \( X, \emptyset, \{a\}, \{b\}, \{a, b\} \) so \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \) is q- \( T_1 \) space.

Theorem 3.8: Every q- \( T_1 \) space is a q- \( T_0 \) space.

Proof: Follows from the definition of q- \( T_1 \) space.

Following example shows that the converse of theorem (3.8) is not true.

Example 3.9: Let \( X = \{a, b, c\} \), \( \tau_1 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\} \), \( \tau_2 = \{X, \emptyset, \{a\}\} \), \( \tau_3 = \{X, \emptyset, \{a, c\}\} \), \( \tau_4 = \{X, \emptyset, \{b\}\} \) all q-open sets are \( X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\} \) so \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \) is q- \( T_0 \) space but it is not q- \( T_1 \) space.

Definition 3.10: A quad topological space \( X \) is said to be q- \( T_2 \) space if and only if for \( x, y \in X, x \neq y \) there exist two disjoint q-open sets \( U, V \) in \( X \) such that \( x \in U \) & \( y \in V \).

Example 3.11: Let \( X = \{a, b, c\} \), \( \tau_1 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\} \), \( \tau_2 = \{X, \emptyset, \{b\}\} \), \( \tau_3 = \{X, \emptyset, \{c\}\} \), \( \tau_4 = \{X, \emptyset, \{a\}\} \) all q-open sets are \( X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \) so \( (X, \tau_1, \tau_2, \tau_3, \tau_4) \) is q- \( T_2 \) space.

Theorem 3.12: Every q- \( T_2 \) space is q- \( T_1 \) space.

Proof: Let \( X \) is q- \( T_2 \) space and let \( x, y \) in \( X \) with \( x \neq y \), so by hypothesis there exist two disjoint q-open, say \( U, V \) such that \( x \in U \) & \( y \in V \) but \( U \cap V = \emptyset \) hence \( x \notin V \) & \( y \notin U \) i.e \( X \) is q- \( T_1 \) space.
**Definition 3.13**: A quad topological space $X$ is said to be q-regular space if and only if for each q-closed set $F$ & each point $x \not\in F$, there exist disjoint q-open sets $U$ & $V$ such that $x \in U$ & $F \subseteq V$.

**Example 3.14**: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}\}$, $\tau_2 = \{X, \emptyset, \{b, c\}\}$, $\tau_3 = \{X, \emptyset, \{a\}, \{b, c\}\}$, $\tau_4 = \{X, \emptyset\}$, all q-open sets are $X, \emptyset, \{a\}, \{b, c\}$ and q-closed sets are $\emptyset, X, \{a\}, \{b, c\}$. So $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is q-regular space.

**Definition 3.15**: A q-regular $T_1$ space is called a $q-T_3$ space.

**Theorem 3.16**: Every $q-T_3$ space is a $q-T_2$ space.

**Proof**: Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a $q-T_3$ space and let $x, y$ be two distinct points in $X$. Now by definition, $X$ is also a $T_1$ space & so $\{x\}$ is a closed set. Also $y \not\in \{x\}$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is a q-regular space, there exist open sets $G$ & $H$ such that $\{x\} \subseteq G$, $y \in H$ & $G \cap H = \emptyset$. Also $\{x\} \subseteq G \implies x \in G$. Thus $x, y$ belong respectively to disjoint open sets $G$ & $H$. According $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is a space.

4. CONCLUSION

In this paper the idea of separation axioms in quad topological space were introduced.

REFERENCES


