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SEPARATION AXIOMS INQUAD TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce separation axioms in quad topological space (q topological spaces) and study some of their properties.

1. INTRODUCTION

J .C. Kelly [2] introduced bitopological spaces in 1963. The study of tri-topological spaces was first initiated by Martin M. Kovar [3] in 2000,where a non empty set X with three topologies is called tri-topolgical spaces. N.F. Hameed & Mohammed Yahya Abid [1] studied separation axioms in tri-topological spaces. D.V. Mukundan [4] introduced the concept on topological structures with four topologies, quad topology (4 – tuple topology) and defined new types of open (closed)sets. In this paper, we use q-open and q-closed sets defined by D.V. Mukundan [4] to explain the concept of separation axioms in quad topological spaces.

2. PRILIMINARIES

Definition 2.1 [4] : Let X be a nonempty set and τ_1, τ_2, τ_3 *and* τ_4 are general topologies on X. Then a subset A of space X is said to be quad-open(q-open) set if $A \subset \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and its complement is said to be q-closed and set X with four topologies called (X, $\tau_1, \tau_2, \tau_3, \tau_4$). q-open sets satisfy all the axioms of topology.
Definition2.2 [4]: A subset of a q-topological space (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is called q- Neighborhood of a point $x \in X$ if and only if there exist q-open sets such that $x \subset X \subset A$.
Note 2.3[4] : We will denote the q-interior (resp. q-closure) of any subset ,say of by q-intA (q-clA),where q-intA is the union of all q-open sets contained in A, and q-clA is the intersection of all q-closed sets containing A.

3. SEPARATION AXIOMS IN QUAD TOPOLOGICAL SPACES

Definition 3.1: A quad (q) topological space X is said to be $q - T_0$ space iff to given any pair of distinct points x,y in X ,there exists a q-open set containing one of the points but not the other .
Example 3.2: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}\}$, $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}\}$, $\tau_3 = \{X, \emptyset, \{a, b\}\}$, $\tau_4 = \{X, \emptyset\}$ all q-open sets are $X, \emptyset, \{a\}, \{a, b\}$ so (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is q-space.
Theorem 3.3: If {x} is q-open for some $x \in X$ then $x \notin q - cl\{y\}$, for all $y \neq x$.

Proof: Let {x} be a q-open for some $x \in X$, then $X-\{x\}$ is q-closed and $x \notin X-\{x\}$. If $x \in q-cl\{y\}$, for some $y \neq x$., then y , x both are in all the q-closed sets containing y, so $x \in X-\{x\}$ which is contradiction, hence $x \notin q-cl\{y\}$.

Theorem 3.4: In any q-topological space X, any distinct points have distinct q-closures.

Proof: Let $x, y \in X$ with $x \neq y$, & let $A=\{x\}^c$ hence $q-cl(A)=A \text{ or } X$. Now if $cl(A)=A$ then A is q-closed so $X-A=\{x\}$ is q-open & not containing y. So by theorem(3.3) $x \notin q-cl(\{y\})$ & $y \in q-cl(\{y\})$ which implies that $q-cl(\{y\})$ and $q-cl(\{x\})$ are distinct. If $q-cl(A)=X$ then A is q-open, hence {x} is q-closed, which mean that $q-cl(\{x\})=\{x\}$ which is not equal to $q-cl(\{y\})$.

Theorem 3.5: In any quad topological space X, if distinct points have distinct *q-T_0* closures then X is q- space .

Proof: Let $x, y \in X$ with $x \neq y$, with $q-cl(\{y\})$ is not equal to $q-cl(\{x\})$, hence there exists $z \in X$ such that $z \in q-cl(\{x\})$, but $z \notin q-cl(\{y\})$ or $z \in q-cl(\{y\})$ but z$\notin$ $q-cl(\{x\})$. Now, without loss of generality, let $z \in q-cl(\{x\})$, but $z \in q-cl(\{y\})$. If $x \in q-cl(\{y\})$, then $q-cl(\{x\})$ is contained in $q-cl(\{y\})$ hence $z \notin q-cl(\{y\})$ which is a contradiction, this mean that $x \notin q-cl(\{y\})$ hence $x \in q-cl(\{y\}^c)$, hence X is $q-T_0$ space.

Definition 3.6: A quad –topological space X is said to be q- T_1 space iff to given any pair of distinct point x & y of X there exist two q-open sets U,V such that $x \in U$, $y \notin U$ & $y \in V$,$x \notin V$.

Example 3.7: Let $X=\{a,b,c\}$, $\tau_1=\{X,\emptyset,\{a\},\{a,b\}\}$, $\tau_2=\{X,\emptyset,\{a\}\}$, $\tau_3=\{X,\emptyset,\{a,b\}\}$, $\tau_4=\{X,\emptyset,\{b\}\}$ all q-open sets are $X,\emptyset,\{a\},\{b\},\{a,b\}$ so (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is q-$T_1$ space.

Theorem 3.8: Every q- T_1space is a $q-T_0$ space.

Proof : Follows from the definition of q- T_1 space.

Following example shows that the converse of theorem (3.8) is not true.

Example 3.9: Let $X=\{a,b,c\}$, $\tau_1=\{X,\emptyset,\{a\},\{a,b\},\{a,c\}\}$, $\tau_2=\{X,\emptyset,\{a\}\}$, $\tau_3=\{X,\emptyset,\{a,c\}\}$, $\tau_4=\{X,\emptyset,\{a,b\}\}$, all q-open sets are $X,\emptyset,\{a\},\{a,b\},\{a,c\}$ so (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is $q-T_0$ space but it is not q- T_1 space.

Definition 3.10: A quad topological space X is said to be q- T_2 space if and only if for $x, y \in X, x \neq y$ there exist two disjoint q-open sets U,V in X such that $x \in U$ & $y \in V$.

Example 3.11: Let $=\{a,b,c\}$, $\tau_1=\{X,\emptyset,\{a\},\{a,b\},\{a,c\}\}$, $\tau_2=\{X,\emptyset,\{b\},\{b,c\}\}$, $\tau_3=\{X,\emptyset,\{c\}\{a,c\}\}$, $\tau_4=\{X,\emptyset,\{a\}\}$ all q-open sets are ,$X,\emptyset,\{a\},\{b\},\{c\},\{a,b\}$ $\{a,c\},\{b,c\}$ so (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is q- T_2 space.

Theorem 3.12: Every q- T_2 space is q- T_1 space .

Proof: Let X is q- T_2 space and let x, y in X with $x \neq y$,so by hypothesis there exist two disjoint q-open ,say U,V such that $x \in U$ & $y \in V$ but $U \cap V = \emptyset$ hence $x \notin V$ & $y \notin U$i.e X is q- T_1 space.

Definition 3.13: A quad topological space X is said to be q-regular space if and only if for each q-closed set F & each point $x \notin F$, there exist disjoint q-open sets U & V such that $x \in U \ \& F \subset V$.

Example 3.14: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}\}$, $\tau_2 = \{X, \emptyset, \{b, c\}\}$ $\tau_3 = \{X, \emptyset, \{a\}, \{b, c\}\}$, $\tau_4 = \{X, \emptyset\}\}$, all q-open sets are $X, \emptyset, \{a\}, \{b, c\}$ and q-closed sets are $\emptyset, X, \{b, c\}, \{a\}$ so (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is q-regular space.

Definition 3.15: A q-regular T_1 space is called a q- T_3 space

Theorem 3.16: Every q- T_3 space is a q- T_2 space.

Proof: Let (X, $\tau_1, \tau_2, \tau_3, \tau_4$) be a q- T_3 space and let x, y be two distinct points in X. Now by definition , X is also a T_1 space & so {x} is a closed set. Also $y \notin \{x\}$. Since (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is a q-regular space, there exist open sets G & H such that $\{x\} \subset G, y \in H \ \& \ G \cap H = \emptyset$. Also $\{x\} \subset G \Longrightarrow x \in G$.Thus x, y belong respectively to disjoint open sets G& H . According (X, $\tau_1, \tau_2, \tau_3, \tau_4$) is a space.

4. CONCLUSION

In this paper the idea of separation axioms in quad topological space were introduced .

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