

SEPARATION AXIOMS IN QUAD TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce separation axioms in quad topological space (q topological spaces) and study some of their properties.

1. INTRODUCTION

J .C. Kelly [2] introduced bitopological spaces in 1963. The study of tri-topological spaces was first initiated by Martin M. Kovar [3] in 2000, where a non empty set X with three topologies is called tri-topological spaces. N.F. Hameed & Mohammed Yahya Abid [1] studied separation axioms in tri-topological spaces. D.V. Mukundan [4] introduced the concept on topological structures with four topologies, quad topology (4 – tuple topology) and defined new types of open (closed)sets. In this paper, we use q-open and q-closed sets defined by D.V. Mukundan [4] to explain the concept of separation axioms in quad topological spaces.

2. PRILIMINARIES

Definition 2.1 [4] : Let X be a nonempty set and τ_1, τ_2, τ_3 and τ_4 are general topologies on X . Then a subset A of space X is said to be quad-open(q-open) set if $A \subset \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and its complement is said to be q-closed and set X with four topologies called $(X, \tau_1, \tau_2, \tau_3, \tau_4)$. q-open sets satisfy all the axioms of topology.

Definition2.2 [4]: A subset of a q-topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is called q- Neighborhood of a point $x \in X$ if and only if there exist q-open sets such that $x \subset X \subset A$.

Note 2.3[4] : We will denote the q-interior (resp. q-closure) of any subset ,say of by q-intA (q-clA), where q-intA is the union of all q-open sets contained in A, and q-clA is the intersection of all q-closed sets containing A.

3. SEPARATION AXIOMS IN QUAD TOPOLOGICAL SPACES

Definition 3.1: A quad (q) topological space X is said to be $q-T_0$ space iff to given any pair of distinct points x, y in X ,there exists a q-open set containing one of the points but not the other .

Example 3.2: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}\}$, $\tau_2 = \{X, \emptyset, \{a\}, \{a, b\}\}$, $\tau_3 = \{X, \emptyset, \{a, b\}\}$, $\tau_4 = \{X, \emptyset\}$ all q-open sets are $X, \emptyset, \{a\}, \{a, b\}$ so $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is q-space.

Theorem 3.3: If $\{x\}$ is q-open for some $x \in X$ then $x \notin q-cl\{y\}$, for all $y \neq x$.

Proof: Let $\{x\}$ be a q-open for some $x \in X$, then $X - \{x\}$ is q-closed and $x \notin X - \{x\}$. If $x \in q-cl\{y\}$, for some $y \neq x$, then y, x both are in all the q-closed sets containing y , so $x \in X - \{x\}$ which is contradiction, hence $x \notin q-cl\{y\}$.

Theorem 3.4: In any q-topological space X , any distinct points have distinct q-closures.

Proof: Let $x, y \in X$ with $x \neq y$, & let $A = \{x\}^c$ hence $q-cl(A) = A$ or X . Now if $cl(A) = A$ then A is q-closed so $X - A = \{x\}$ is q-open & not containing y . So by theorem(3.3) $x \notin q-cl(\{y\})$ & $y \in q-cl(\{y\})$ which implies that $q-cl(\{y\})$ and $q-cl(\{x\})$ are distinct. If $q-cl(A) = X$ then A is q-open, hence $\{x\}$ is q-closed, which mean that $q-cl(\{x\}) = \{x\}$ which is not equal to $q-cl(\{y\})$.

Theorem 3.5: In any quad topological space X , if distinct points have distinct $q-T_0$ closures then X is q-space .

Proof: Let $x, y \in X$ with $x \neq y$, with $q-cl(\{y\})$ is not equal to $q-cl(\{x\})$, hence there exists $z \in X$ such that $z \in q-cl(\{x\})$, but $z \notin q-cl(\{y\})$ or $z \in q-cl(\{y\})$ but $z \notin q-cl(\{x\})$. Now, without loss of generality, let $z \in q-cl(\{x\})$, but $z \in q-cl(\{y\})$. If $x \in q-cl(\{y\})$, then $q-cl(\{x\})$ is contained in $q-cl(\{y\})$ hence $z \notin q-cl(\{y\})$. which is a contradiction, this mean that $x \notin q-cl(\{y\})$ hence $x \in q-cl(\{y\}^c)$, hence X is $q-T_0$ space.

Definition 3.6: A quad -topological space X is said to be $q-T_1$ space iff to given any pair of distinct point x & y of X there exist two q-open sets U, V such that $x \in U, y \notin U$ & $y \in V, x \notin V$.

Example 3.7: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \emptyset, \{a\}\}$, $\tau_3 = \{X, \emptyset, \{a, b\}\}$, $\tau_4 = \{X, \emptyset, \{b\}\}$ all q-open sets are $X, \emptyset, \{a\}, \{b\}, \{a, b\}$ so $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is $q-T_1$ space.

Theorem 3.8: Every $q-T_1$ space is a $q-T_0$ space.

Proof : Follows from the definition of $q-T_1$ space.

Following example shows that the converse of theorem (3.8) is not true.

Example 3.9: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$, $\tau_2 = \{X, \emptyset, \{a\}\}$, $\tau_3 = \{X, \emptyset, \{a, c\}\}$, $\tau_4 = \{X, \emptyset, \{a, b\}\}$, all q-open sets are $X, \emptyset, \{a\}, \{a, b\}, \{a, c\}$ so $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is $q-T_0$ space but it is not $q-T_1$ space.

Definition 3.10: A quad topological space X is said to be $q-T_2$ space if and only if for $x, y \in X, x \neq y$ there exist two disjoint q-open sets U, V in X such that $x \in U$ & $y \in V$.

Example 3.11: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}\}$, $\tau_2 = \{X, \emptyset, \{b\}, \{b, c\}\}$, $\tau_3 = \{X, \emptyset, \{c\}, \{a, c\}\}$, $\tau_4 = \{X, \emptyset, \{a\}\}$ all q-open sets are $X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}$ so $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is $q-T_2$ space.

Theorem 3.12: Every $q-T_2$ space is $q-T_1$ space .

Proof: Let X is $q-T_2$ space and let x, y in X with $x \neq y$, so by hypothesis there exist two disjoint q-open ,say U, V such that $x \in U$ & $y \in V$ but $U \cap V = \emptyset$ hence $x \notin V$ & $y \notin U$. i.e X is $q-T_1$ space.

Definition 3.13: A quad topological space X is said to be q-regular space if and only if for each q-closed set F & each point $x \notin F$, there exist disjoint q-open sets U & V such that $x \in U$ & $F \subset V$.

Example 3.14: Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}\}$, $\tau_2 = \{X, \emptyset, \{b, c\}\}$, $\tau_3 = \{X, \emptyset, \{a\}, \{b, c\}\}$, $\tau_4 = \{X, \emptyset\}$, all q-open sets are $X, \emptyset, \{a\}, \{b, c\}$ and q-closed sets are $\emptyset, X, \{b, c\}, \{a\}$ so $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is q-regular space.

Definition 3.15: A q-regular T_1 space is called a $q-T_3$ space

Theorem 3.16: Every $q-T_3$ space is a $q-T_2$ space.

Proof: Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a $q-T_3$ space and let x, y be two distinct points in X . Now by definition, X is also a T_1 space & so $\{x\}$ is a closed set. Also $y \notin \{x\}$. Since $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is a q-regular space, there exist open sets G & H such that $\{x\} \subset G, y \in H$ & $G \cap H = \emptyset$. Also $\{x\} \subset G \Rightarrow x \in G$. Thus x, y belong respectively to disjoint open sets G & H . According $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ is a space.

4. CONCLUSION

In this paper the idea of separation axioms in quad topological space were introduced .

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