t-Intuitionistic Fuzzy Subalgebra of BG-Algebras

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Abstract. The aim of this paper is to introduced the notion of t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra of BG-algebras. We state and prove some theorems in t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra in BG-algebras. The homomorphic image and inverse image are investigated in both t-intuitionistic fuzzy subalgebra and normal subalgebras.

Introduction

In 1966, Imai and Iseki [6] introduced the two classes of abstract algebras, viz., BCK-algebras and BCI-algebras. It is known that the class of BCK-algebra is a proper subclass of the class of BCI-algebras. Neggers and Kim [8] introduced a new concept, called B-algebras, which are related to several classes of algebras such as BCI/BCK-algebras. Kim and Kim [7] introduced the notion of BG-algebra which is a generalization of B-algebra. The concept of intuitionistic fuzzy subset (IFS) was introduced by Atanassov [5] in 1983, which is a generalization of the notion of fuzzy sets. The concept of fuzzy subalgebras of BG-algebras was introduced by Ahn and Lee in [1]. The study of intuitionistic fuzzification of subalgebras and ideals of BG-algebras is done by Senapati et. al in [9]. The idea of t-intuitionistic fuzzy sets in fuzzy subgroups and fuzzy subrings is introduced by Sharma in [10, 11]. Here in this paper, we introduced the notion of t-intuitionistic fuzzy sets in fuzzy subalgebra and fuzzy normal subalgebras of BG-algebras and study their properties.

Preliminaries

Definition 0.1 ([1]) A BG-algebra is a non-empty set X with a constant Ā′ 0′ and a binary operation Ā* Ā satisfies the following axioms:

(i) x * x = 0,
(ii) x * 0 = x,
(iii) (x * y) * (0 * y) = x, ∀ x, y ∈ X.

For brevity, we also call X a BG-algebra. We can define a partial ordering “≤” on X by x ≤ y iff x * y = 0

Definition 0.2 ([1]) A non-empty subset S of a BG-algebra X is called a subalgebra of X if x * y ∈ S, for all x, y ∈ S.
Definition 0.3 Let X and Y be two non-empty sets and \( f : X \rightarrow Y \) be a mapping. Let A and B be IFS’s of X and Y respectively. Then the image of A under the map f is denoted by \( f(A) \) and is defined by

\[
\nu_{f(A)}(y) = \begin{cases} 
\nu_A(x) : x \in f^{-1}(y) \\
0 & \text{otherwise}
\end{cases}
\]

\[
\mu_{f(A)}(y) = \begin{cases} 
\mu_A(x) : x \in f^{-1}(y) \\
\varnothing & \text{otherwise}
\end{cases}
\]

also pre image of B under f is denoted by \( f^{-1}(B) \) and is defined as

\[
f^{-1}(B) = \{ x \in X : f(x) \in B \}; \forall x \in X
\]

Remark Note that \( \nu_A(x) \leq \nu_{f(A)}(f(x)) \) and \( \nu_A(x) \geq \nu_{f(A)}(f(x)) \) \( \forall x \in X \) however equality hold when the map f is bijective.

Definition 0.4 ([2, 3]) An intuitionistic fuzzy set (IFS) \( A \) in a non empty set X is an object of the form

\[
A = \{ x, \mu_A(x), \nu_A(x) > |x \in X \}
\]

where \( \mu_A : X \rightarrow [0, 1] \) and \( \nu_A : X \rightarrow [0, 1] \) with the condition

\[
0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X.
\]

The numbers \( \mu_A(x) \) and \( \nu_A(x) \) denote respectively the degree of membership and the degree of non membership of the element x in the set A. For the sake of simplicity we shall use the symbol \( (\mu_A, \nu_A) \) for the intuitionistic fuzzy set \( A = \{ x, \mu_A(x), \nu_A(x) > |x \in X \} \). The function \( \pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \) for all \( x \in X \). is called the degree of uncertainty of \( x \in A \). The class of IFS’s on a universe X is denoted by \( IFS(X) \).

Definition 0.5 ([2, 3]) If \( A = \{ x, \mu_A(x), \nu_A(x) > |x \in X \} \) and \( B = \{ x, \mu_B(x), \nu_B(x) > |x \in X \} \) be any two IFS of a set X then

(1) \( A \subseteq B \) iff for all \( x \in X, \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \)

(2) \( A \cap B = \{ x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \} \)

(3) \( A \cup B = \{ x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \} \)

Definition 0.6 ([4]) For any IFS \( A = \{ x, \mu_A(x), \nu_A(x) > |x \in X \} \) of X and \( \alpha \in [0, 1] \), the operator \( \Box : IFS(X) \rightarrow IFS(X), \Diamond : IFS(X) \rightarrow IFS(X) \) are defined as

(i) \( \Box(A) = \{ x, \mu_A(x), 1 - \alpha \mu_A(x) > |x \in X \} \) is called necessity operator

(ii) \( \Diamond(A) = \{ x, 1 - \nu_A(x), \nu_A(x) > |x \in X \} \) is called possibility operator

(iii) \( D_\alpha(A) = \{ x, \mu_A(x) + \alpha \pi_A(x), \nu_A(x) + (1 - \alpha) \pi_A(x) > |x \in X \} \) is called \( \alpha \)– Model operator. Clearly \( \Box(A) \subseteq A \subseteq \Diamond(A) \) and the equality hold, when A is a fuzzy set also \( D_0(A) = \Box(A) \) and \( D_1(A) = \Diamond(A) \). Therefore the \( \alpha \)– Model operator \( D_\alpha(A) \) is an extension of necessity operator \( \Box(A) \) and possibility operator \( \Diamond(A) \).

Definition 0.7 ([4]) For any IFS \( A = \{ x, \mu_A(x), \nu_A(x) > |x \in X \} \) of X and for any \( \alpha, \beta \in [0, 1] \) such that \( \alpha + \beta \leq 1 \), the \((\alpha, \beta)\)– model operator \( F_{\alpha, \beta} : IFS(X) \rightarrow IFS(X) \) is defined as

\[
F_{\alpha, \beta}(A) = \{ x, \mu_A(x) + \alpha \pi_A(x), \nu_A(x) + \beta \pi_A(x) > |x \in X \}, \text{ where } \pi_A(x) = 1 - \mu_A(x) - \nu_A(x)
\]

for all \( x \in X \). Therefore we can write

\[
F_{\alpha, \beta}(A) = \{ x, \mu_{F_{\alpha, \beta}}(A)(x), \nu_{F_{\alpha, \beta}}(A)(x) \}
\]

where \( \mu_{F_{\alpha, \beta}}(x) = \mu_A(x) + \alpha \pi_A(x) \) and \( \nu_{F_{\alpha, \beta}}(A)(x) = \nu_A(x) + \beta \pi_A(x) \). Clearly, \( F_{0,1}(A) = \Box(A), F_{1,0}(A) = \Diamond(A) \) and \( F_{\alpha, 1-\alpha}(A) = D_\alpha(A) \)

Definition 0.8 ([9]) An intuitionistic fuzzy set \( A = (\mu_A, \nu_A) \) of a BG-algebra of X is said to be an intuitionistic fuzzy subalgebra of X if

(i) \( \mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\} \)

(ii) \( \nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\} \)

\( \forall x, y \in X \).
**Definition 0.9** ([7]) An IFS $A$ of a BG-algebra $X$ is said to be an IF normal subalgebra of $X$ if

(i) $\mu_A((x * a) * (y * b)) \geq \min\{\mu_A(x * y), \mu_A(a * b)\}$,

(ii) $\nu_A((x * a) * (y * b)) \leq \max\{\nu_A(x * y), \nu_A(a * b)\}$, $\forall x, y \in X$.

**Definition 0.10** ([11]) Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of BG-algebra $X$. Let $t \in [0, 1]$. Then the intuitionistic fuzzy set of $A'$ of $X$ is called $t$-intuitionistic fuzzy subset ($t$-IFS) of $X$ w.r.t $A$ and is defined by $A' = \{< x, \mu_{A'}(x), \nu_{A'}(x) > | x \in X \} = < \mu_{A'}, \nu_{A'} >$ where $\mu_{A'}(x) = \min\{\mu_A(x), t\}$ and $\nu_{A'} = \max\{\nu_A(x), 1 - t\}, \forall x \in X$.

**Remark 0.11** ([11]) Let $A^t = < \mu_A^t, \nu_A^t >$ and $B^t = < \mu_B^t, \nu_B^t >$ be two $t$-intuitionistic fuzzy subsets of BG-algebra $X$, then $(A \cap B)^t = A^t \cap B^t$.

**Remark 0.12** ([11]) Let $f : X \to Y$ be a mapping. Let $A$ and $B$ are two IFS of $X$ and $Y$ respectively, then

(i) $f^{-1}(B^t) = (f^{-1}(B))^t$ (ii) $f(A^t) = (f(A))^t \forall t \in [0, 1]$

**Definition 0.13** Let $A^t = < \mu_A^t, \nu_A^t >$ and $B^t = < \mu_B^t, \nu_B^t >$ be two $t$-intuitionistic fuzzy subsets of BG-algebra $X$. Then their cartesian product $A^t \times B^t = < \mu_{A^t \times B^t}, \nu_{A^t \times B^t} >$ is defined by

$\mu_{A^t \times B^t}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$

$\nu_{A^t \times B^t}(x, y) = \max\{\nu_A(x), \nu_B(y)\}$, $\forall x, y \in X$.

### t-Intuitionistic Fuzzy Subalgebra BG-algebra

Now onwards, let $X$ denote a BG-algebra unless otherwise stated.

**Definition 0.14** Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of BG-algebra $X$. Let $t \in [0, 1]$ then $A$ is called $t$-intuitionistic fuzzy subalgebra (t-IFSA) of $X$ if $A^t$ is IFSA of $X$ i.e. if $A^t$ satisfies following conditions:

\[
\begin{align*}
\mu_A^t(x * y) &\geq \min\{\mu_A(x), \mu_A(y)\} \\
\nu_A^t(x * y) &\leq \max\{\nu_A(x), \nu_A(y)\}
\end{align*}
\]

**Theorem 0.15** If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subalgebra BG-algebra $X$, then $A$ is also $t$-intuitionistic fuzzy subalgebra of $X$.

**Proof.** Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subalgebra BG-algebra $X$, therefore

\[
\begin{align*}
\mu_A(x * y) &\geq \min\{\mu_A(x), \mu_A(y)\} \\
\nu_A(x * y) &\leq \max\{\nu_A(x), \nu_A(y)\}, \forall x, y \in X.
\end{align*}
\]

Now, $\mu_A^t(x * y) = \min\{\mu_A(x * y), t\}$

$\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, t\}$

$= \min\{\min(\mu_A(x), t), \min(\mu_A(y), t)\}$

$= \min\{\mu_A(x), \mu_A(y)\}$

$\Rightarrow \mu_A^t(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$

Similarly we can show

$\nu_A^t(x * y) \leq \max\{\nu_A(x), \nu_A(y)\}$

Hence $A$ is also $t$-intuitionistic fuzzy subalgebra BG-algebra $X$. 
Remark 0.16 The converse of above Theorem is not true.

Example 1. Consider a $BG$-algebra $X = \{0, 1, 2\}$ with the following Cayley table:

Table 1: Example of intuitionistic fuzzy $BG$-subalgebra.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X \}$ given by $\mu_A(0) = 0.4$, $\mu_A(1) = 0.5, \mu_A(2) = 0.3$ and $\nu_A(0) = 0.5, \nu_A(1) = 0.4, \nu_A(2) = 0.6$. Since $\mu_A(0) = 0.4 \notin \min\{\mu_A(1), \mu_A(1)\}$. Therefore $A$ is not an intuitionistic fuzzy $BG$-subalgebra of $X$. Take $t = 0.2$. Then $\mu_A(x) > t$ for all $x \in X$ and also $\nu_A(x) < 1 - t$ for all $x \in X$.

Therefore $\mu_A(x \ast y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x \ast y) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all $x \in X$. Hence $A$ is $t$-intuitionistic fuzzy subalgebra of $X$.

Theorem 0.17 If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set of $BG$-algebra $X$ and let $t < \min\{p, 1 - q\}$, where $p = \min\{\mu_A(x) | x \in X\}$ and $q = \max\{\nu_A(x) | x \in X\}$ then $A$ is also $t$-intuitionistic fuzzy subalgebra $BG$-algebra $X$.

Proof. Since $t < \min\{p, 1 - q\}$

$t < \min\{p, 1 - q\}$

$\Rightarrow$ $p > t$ and $1 - q > t$

$\Rightarrow$ $p > t$ and $q < 1 - t$

$\Rightarrow$ $\min\{\mu_A(x) | x \in X\} > t$ and $\max\{\nu_A(x) | x \in X\} < 1 - t$

$\Rightarrow$ $\mu_A(x) > t, \forall \ x \in X$ and $\nu_A(x) < 1 - t, \forall \ x \in X$

Therefore $\mu_A(x \ast y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x \ast y) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all $x \in X$. Hence $A$ is $t$-intuitionistic fuzzy subalgebra of $X$.

Theorem 0.18 Any IF set of $BG$-algebra $X$ can be realised as $t$-intuitionistic fuzzy subalgebra $X$.

Proof. It follows from Theorem 0.17 and Theorem 0.15.

Theorem 0.19 The intersection of two $t$-intuitionistic fuzzy subalgebra $BG$-algebra $X$ is also a $t$-intuitionistic fuzzy subalgebra of $X$. 

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Proof. Let \( x, y \in X \). Then
\[
\mu_{(A \cap B)^t}(x \ast y) = \min \{ \mu_{(A \cap B)}(x \ast y), t \}
\]
\[
\geq \min \{ \min \{ \mu_A(x \ast y), \mu_A(x \ast y) \}, t \}
\]
\[
= \min \{ \min \{ \mu_A(x \ast y), t \}, \min \{ \mu_B(x \ast y), t \} \}
\]
\[
= \min \{ \mu_A(x \ast y), \mu_B(x \ast y) \}
\]
\[
\geq \min \{ \min \{ \mu_A(x), \mu_B(y) \}, \min \{ \mu_B(x), \mu_B(y) \} \}
\]
\[
= \min \{ \min \{ \mu_A(x), \mu_B(x) \}, \min \{ \mu_A(y), \mu_B(y) \} \}
\]
\[
= \min \{ \mu_{(A \cap B)^t}(x), \mu_{(A \cap B)^t}(y) \}
\]
\[
\Rightarrow \mu_{(A \cap B)^t}(x \ast y) \geq \min \{ \mu_{(A \cap B)^t}(x), \mu_{(A \cap B)^t}(y) \}
\]
Similarly we can show that
\[
\nu_{(A \cap B)^t}(x \ast y) \leq \max \{ \nu_{(A \cap B)^t}(x), \nu_{(A \cap B)^t}(y) \}
\]

**Theorem 0.20** The intersection of any number of t-intuitionistic fuzzy subalgebra \( BG \)-algebra \( X \) is also a t-intuitionistic fuzzy subalgebra of \( X \).

**Theorem 0.21** For every t-intuitionistic fuzzy subalgebra \( A^t \) of \( X \), the following properties hold
(i) \( \mu_{A^t}(0) \geq \mu_{A^t}(x) \)
(ii) \( \nu_{A^t}(0) \leq \nu_{A^t}(x), \forall x \in X \).

Proof. We have \( \mu_{A^t}(0) = \mu_{A^t}(x \ast x) \geq \min \{ \mu_{A^t}(x), \mu_{A^t}(x) \} = \mu_{A^t}(x) \)
and \( \nu_{A^t}(0) = \nu_{A^t}(x \ast x) \leq \max \{ \nu_{A^t}(x), \nu_{A^t}(x) \} = \nu_{A^t}(x) \)

**Theorem 0.22** If \( A \) be IF subalgebra of \( BG \)-algebra \( X \), then \( \Box A \), \( \diamond A \) and \( F_{a,b}(A) \) are also t-intuitionistic fuzzy subalgebra of \( X \).

Proof. Here \( A \) be IF subalgebra of \( BG \)-algebra \( X \), By Theorem 0.15 \( A \) is also t-intuitionistic fuzzy subalgebra of \( X \).
\[
\mu_{A^t}(x \ast y) \geq \min \{ \mu_{A^t}(x), \mu_{A^t}(y) \} \quad (1)
\]
\[
\nu_{A^t}(x \ast y) \leq \max \{ \nu_{A^t}(x), \nu_{A^t}(y) \} \quad \forall x, y \in X. \quad (2)
\]
Now \( \Box A^t = \{ < x, \mu_{A^t}(x), 1 - \mu_{A^t}(x) | x \in X \} = \{ < x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) | x \in X \} \)
\( \diamond A^t = \{ < x, 1 - \nu_{A^t}(x), \mu_{A^t}(x) | x \in X \} = \{ < x, \overline{\nu_{A^t}}(x), \mu_{A^t}(x) | x \in X \} \)

Now
\[
\overline{\nu_{A^t}}(x \ast y) = 1 - \mu_{A^t}(x \ast y)
\]
\[
\leq 1 - \min \{ \mu_{A^t}(x), \mu_{A^t}(y) \} \quad \text{By (1)}
\]
\[
= \max \{ 1 - \mu_{A^t}(x), 1 - \mu_{A^t}(y) \}
\]
\[
= \max \{ \overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y) \}
\]
\[
\Rightarrow \overline{\nu_{A^t}}(x \ast y) \leq \max \{ \overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y) \} \quad (3)
\]
Hence by Eqn (1) and (3) \( \Box A^t = \{ < x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) | x \in X \} \) is t-intuitionistic fuzzy subalgebra of \( X \).
Similarly we can show that
\[ \Diamond A' = \{ x, \mu_{A'}(x), \nu_{A'}(x) \mid x \in X \} \text{ is t-intuitionistic fuzzy subalgebra of } X. \]

Again, we have \( F_{\alpha,\beta}(A) = < \mu_{F_{\alpha,\beta}(A)}, \nu_{F_{\alpha,\beta}(A)} > \) let \( x, y \in X \), then \( F_{\alpha,\beta}(x \ast y) = (\mu_{F_{\alpha,\beta}(A)}(x \ast y), \nu_{F_{\alpha,\beta}(A)}(x \ast y)) \) where \( \mu_{F_{\alpha,\beta}(A)}(x \ast y) = \mu_A(x \ast y) + \alpha \pi_A(x \ast y) \) and \( \nu_{F_{\alpha,\beta}(A)}(x \ast y) = \nu_A(x \ast y) + \beta \pi_A(x \ast y) \)

\[
\mu_{F_{\alpha,\beta}A'}(x \ast y) \\
= \mu_A(x \ast y) + \alpha \pi_A(x \ast y) \\
= \mu_A(x \ast y) + \alpha(1 - \mu_A(x \ast y) - \nu_A(x \ast y)) \\
\geq \alpha + (1 - \alpha)\mu_A(x \ast y) - \alpha \nu_A(x \ast y) \\
\geq \alpha + (1 - \alpha)\min(\mu_A(x), \mu_A(y)) - \alpha \max(\nu_A(x), \nu_A(y)) \quad \text{By (1)} \\
\geq \alpha \{(1 - \max(\nu_A(x), \nu_A(y))) + (1 - \alpha)\min(\mu_A(x), \mu_A(y))\} \\
\geq \alpha \{(1 - \nu_A(x)) + (1 - \alpha)\mu_A(x), \alpha(1 - \nu_A(y)) + (1 - \alpha)\mu_A(y)\} \\
\geq \min(\{\mu_A(x) + \alpha(1 - \mu_A(x) - \nu_A(x)), \mu_A(y) + \alpha(1 - \mu_A(y) - \nu_A(y))\}) \\
\geq \min(\{\mu_{F_{\alpha,\beta}A'}(x), \mu_{F_{\alpha,\beta}A'}(y)\}) \\
\therefore \quad \mu_{F_{\alpha,\beta}A'}(x \ast y) \geq \min(\{\mu_{F_{\alpha,\beta}A'}(x), \mu_{F_{\alpha,\beta}A'}(y)\})
\]

Similarly we can prove that
\( \nu_{F_{\alpha,\beta}A'}(x \ast y) \leq \max(\nu_{F_{\alpha,\beta}A'}(x), \nu_{F_{\alpha,\beta}A'}(y)) \)

Hence \( F_{\alpha,\beta}(A) \) is t-intuitionistic fuzzy subalgebra of \( X \).

**Theorem 0.23** Cartesian product of two t-intuitionistic fuzzy subalgebra of \( X \) is again a t-intuitionistic fuzzy subalgebra of \( X \times X \).

**Proof.** Let \( A' = < \mu_{A'}, \nu_{A'} > \) and \( B' = < \mu_{B'}, \nu_{B'} > \) be two t-intuitionistic fuzzy subalgebra of BG-algebra \( X \)

Then their cartesian product \( A' \times B' = < \mu_{A' \times B'}, \nu_{A' \times B'} > \), where
\[
\mu_{A' \times B'}(x, y) = \min\{\mu_{A'}(x), \mu_{B'}(y)\} \\
\nu_{A' \times B'}(x, y) = \max\{\nu_{A'}(x), \nu_{B'}(y)\} \quad \forall x, y \in X.
\]

Also
\[
\mu_{A'}(x \ast y) \geq \min(\mu_{A'}(x), \mu_{A'}(y)) \quad \text{(4)} \\
\nu_{A'}(x \ast y) \leq \max(\nu_{A'}(x), \nu_{A'}(y)) \quad \forall x, y \in X. \quad \text{(5)}
\]

\[
\mu_{A' \times B'}((x_1, y_1) \ast (x_2, y_2)) = \mu_{A' \times B'}(x_1 \ast x_2, y_1 \ast y_2) \\
= \min\{\mu_{A'}(x_1 \ast x_2), \mu_{B'}(y_1 \ast y_2)\} \\
\geq \min\{\min(\mu_{A'}(x_1), \mu_{A'}(x_2)), \min(\mu_{B'}(y_1), \mu_{B'}(y_2))\} \\
= \min\{\min(\mu_{A'}(x_1), \mu_{B'}(y_1)), \min(\mu_{A'}(x_2), \mu_{B'}(y_2))\} \\
= \min\{\mu_{A' \times B'}((x_1, y_1), \mu_{A' \times B'}((x_2, y_2))\}
\]

\( \Rightarrow \mu_{A' \times B'}((x_1, y_1) \ast (x_2, y_2)) \geq \min(\mu_{A' \times B'}((x_1, y_1), \mu_{A' \times B'}((x_2, y_2))\}

Similarly we can show
\( \nu_{A' \times B'}((x_1, y_1) \ast (x_2, y_2)) \leq \max(\nu_{A' \times B'}((x_1, y_1), \nu_{A' \times B'}((x_2, y_2))\}

Corollary 0.24 If \( A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle \) and \( B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle \) be two t-intuitionistic fuzzy subalgebra of BG-algebra \( X \). Then \( \square(A^t \times B^t) \cap (A^t \times B^t) \), \( F_{\alpha, \beta}(A^t \times B^t) \) are also t-intuitionistic fuzzy subalgebra of \( X \times X \).

Proof. Same as Corollary 0.24.

Theorem 0.25 If \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy normal subalgebra BG-algebra \( X \), then \( A \) is also t-intuitionistic fuzzy normal subalgebra of \( X \).

Proof. Since \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy normal subalgebra BG-algebra \( X \), therefore

\[
(i) \mu_A((x \ast a) \ast (y \ast b)) \geq \min\{\mu_A(x \ast y), \mu_A(a \ast b)\} \\
(ii) \nu_A((x \ast a) \ast (y \ast b)) \leq \max\{\nu_A(x \ast y), \nu_A(a \ast b)\}, \forall x, y \in X.
\]

Now, \( \mu_{A^t}((x \ast a) \ast (y \ast b)) = \min\{\mu_A((x \ast a) \ast (y \ast b)), t\} \geq \min\{\min\{\mu_A(x \ast y), \mu_A(a \ast b)\}, t\} = \min\{\min\{\mu_A(x \ast y), t\}, \min\{\mu_A(a \ast b), t\}\} = \min\{\mu_{A^t}(x \ast y), \mu_{A^t}(a \ast b)\} \Rightarrow \mu_{A^t}((x \ast a) \ast (y \ast b)) \geq \min\{\mu_{A^t}(x \ast y), \mu_{A^t}(a \ast b)\}

Similarly we can show that

\[
\nu_{A^t}((x \ast a) \ast (y \ast b)) \leq \max\{\nu_{A^t}(x \ast y), \nu_{A^t}(a \ast b)\}
\]

Hence \( A \) is also t-intuitionistic fuzzy normal subalgebra BG-algebra \( X \).

Remark 0.26 The converse of above Theorem is not true.

Theorem 0.27 If \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy set of BG-algebra \( X \) and let \( t < \min\{p, 1-q\} \), where \( p = \min\{\mu_A(x) \mid x \in X\} \) and \( q = \max\{\nu_A(x) \mid x \in X\} \) then \( A \) is also t-intuitionistic fuzzy normal subalgebra BG-algebra \( X \).

Proof. Same as Theorem 0.17.

Theorem 0.28 The intersection of two t-intuitionistic fuzzy normal subalgebra BG-algebra \( X \) is also a t-intuitionistic fuzzy normal subalgebra of \( X \).

Proof. Same as Theorem 0.19.

Theorem 0.29 If \( A \) be IF normal subalgebra of BG-algebra \( X \), then \( \square A, \triangledown A \) and \( F_{\alpha, \beta}(A) \) are also t-intuitionistic fuzzy normal subalgebra of \( X \).

Proof. Same as Theorem 0.22.

Theorem 0.30 Cartesian product of two t-intuitionistic fuzzy normal subalgebra of \( X \) is again a t-intuitionistic fuzzy normal subalgebra of \( X \times X \).

Proof. Same as Theorem 0.23.

Corollary 0.31 If \( A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle \) and \( B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle \) be two t-intuitionistic fuzzy normal subalgebra of BG-algebra \( X \). Then \( \square(A^t \times B^t) \), \( \triangledown(A^t \times B^t) \), \( F_{\alpha, \beta}(A^t \times B^t) \) are also t-intuitionistic fuzzy normal subalgebra of \( X \times X \).

Proof. Same as Corollary 0.24.
Homomorphism of t-intuitionistic fuzzy subalgebra $BG$-algebra

**Definition 0.32** Let $X$ and $Y$ be two $BG$-algebras, then a mapping $f : X \rightarrow Y$ is said to be homomorphism if $f(x \ast y) = f(x) \ast f(y), \forall x, y \in X$.

**Theorem 0.33** Let $f : X \rightarrow Y$ be a homomorphism of $BG$-algebras, If $A$ be a t-intuitionistic fuzzy subalgebra of $Y$, then $f^{-1}(A)$ is t-intuitionistic fuzzy subalgebra $X$.

**Proof.** Let $y_1, y_2 \in Y$. Since $f$ is onto, therefore there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$.

Let $A$ be a t-intuitionistic fuzzy normal subalgebra $X$. Then $f(A)$ is a t-intuitionistic fuzzy subalgebra in $Y$.

**Theorem 0.35** Let $f : X \rightarrow Y$ be a onto homomorphism of $BG$-algebras, If $A$ be t-intuitionistic fuzzy subalgebra $X$. Then $f(A)$ is t-intuitionistic fuzzy subalgebra of $Y$.

**Proof.** Now, $\mu_{f^{-1}(A)}(x \ast y)$

$$= \mu_{A}(x \ast y)$$

$$= \mu_{A}(f(x) \ast f(y))$$

$$\geq \min \{ \mu_{A}(f(x)), \mu_{A}(f(y)) \}$$

[Since $A$ is t-IF subalgebra of $Y$]

$$= \min \{ \mu_{f^{-1}(A)}(x), \mu_{f^{-1}(A)}(y) \}$$

Therefore $\mu_{f^{-1}(A)}(x \ast y) \geq \min \{ \mu_{f^{-1}(A)}(x), \mu_{f^{-1}(A)}(y) \}$

Similarly we can show that $\nu_{f^{-1}(A)}(x \ast y) \leq \max \{ \nu_{f^{-1}(A)}(x), \nu_{f^{-1}(A)}(y) \}$

Hence, $f^{-1}(A) = (f^{-1}(A))^t$ is t-intuitionistic fuzzy subalgebra $X$.

Hence $f(A)$ is t-intuitionistic fuzzy subalgebra of $Y$. 
Theorem 0.36 Let \( f: X \to Y \) be a onto homomorphism of BG-algebras, If \( A \) be t-intuitionistic fuzzy normal subalgebra \( X \), then \( f(A) \) is t-intuitionistic fuzzy normal subalgebra of \( Y \).

References


