

## t- Intuitionistic Fuzzy Subalgebra of *BG*-Algebras

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**Keywords:** *BG*-algebra, Subalgebra, t-Intuitionistic Fuzzy Set, t-Intuitionistic Fuzzy Subalgebra and t-Intuitionistic Fuzzy Normal Subalgebra.

**Abstract.** The aim of this paper is to introduced the notion of t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra of *BG*-algebras. We state and prove some theorems in t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra in *BG*-algebras. The homomorphic image and inverse image are investigated in both t-intuitionistic fuzzy subalgebra and normal subalgebras.

### Introduction

In 1966, Imai and Iseki [6] introduced the two classes of abstract algebras, viz., *BCK*-algebras and *BCI*-algebras. It is known that the class of *BCK*-algebra is a proper subclass of the class of *BCI*-algebras. Neggers and Kim [8] introduced a new concept, called *B*-algebras, which are related to several classes of algebras such as *BCI/BCK*-algebras. Kim and Kim [7] introduced the notion of *BG*-algebra which is a generalization of *B*-algebra. The concept of intuitionistic fuzzy subset (IFS) was introduced by Atanassov [5] in 1983, which is a generalization of the notion of fuzzy sets. The concept of fuzzy subalgebras of *BG*-algebras was introduced by Ahn and Lee in [1]. The study of intuitionistic fuzzification of subalgebras and ideals of *BG*-algebras is done by Senapati et. al in [9]. The idea of t-intuitionistic fuzzy sets in fuzzy subgroups and fuzzy subrings is introduced by Sharma in [10, 11]. Here in this paper, we introduced the notion of t-intuitionistic fuzzy sets in fuzzy subalgebra and fuzzy normal subalgebras of *BG*-algebras and study their properties.

### Preliminaries

**Definition 0.1** ([1]) A *BG*-algebra is a non-empty set  $X$  with a constant '0' and a binary operation '\*' satisfying the following axioms:

- (i)  $x * x = 0$ ,
- (ii)  $x * 0 = x$ ,
- (iii)  $(x * y) * (0 * y) = x, \forall x, y \in X$ .

For brevity, we also call  $X$  a *BG*-algebra. We can define a partial ordering " $\leq$ " on  $X$  by  $x \leq y$  iff  $x * y = 0$

**Definition 0.2** ([1]) A non-empty subset  $S$  of a *BG*-algebra  $X$  is called a subalgebra of  $X$  if  $x * y \in S$ , for all  $x, y \in S$ .

**Definition 0.3** Let  $X$  and  $Y$  be two non empty sets and  $f : X \rightarrow Y$  be a mapping. Let  $A$  and  $B$  be IFS's of  $X$  and  $Y$  respectively. Then the image of  $A$  under the map  $f$  is denoted by  $f(A)$  and is

$$\text{defined by } f(A)(y) = (\mu_{f(A)}y, \nu_{f(A)}y), \text{ where } \mu_{f(A)}(y) = \begin{cases} \bigvee \{\mu_A(x) : x \in f^{-1}(y)\} \\ 0 \end{cases} \quad \nu_{f(A)}(y) = \begin{cases} \bigwedge \{\nu_A(x) : x \in f^{-1}(y)\} \\ 1 \end{cases} \text{ otherwise}$$

also pre image of  $B$  under  $f$  is denoted by  $f^{-1}(B)$  and is defined as  $f^{-1}(B)(x) = (\mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x)) = (\mu_B(f(x)), \nu_B(f(x))) ; \forall x \in X$

*Remark* Note that  $\mu_A(x) \leq \mu_{f(A)}(f(x))$  and  $\nu_A(x) \geq \nu_{f(A)}(f(x)) \quad \forall x \in X$  however equality hold when the map  $f$  is bijective.

**Definition 0.4** ([2, 3]) An intuitionistic fuzzy set (IFS)  $A$  in a non empty set  $X$  is an object of the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$  where  $\mu_A : X \rightarrow [0, 1]$  and  $\nu_A : X \rightarrow [0, 1]$  with the condition  $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$ . The numbers  $\mu_A(x)$  and  $\nu_A(x)$  denote respectively the degree of membership and the degree of non membership of the element  $x$  in the set  $A$ . For the sake of simplicity we shall use the symbol  $A = (\mu_A, \nu_A)$  for the intuitionistic fuzzy set  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$ . The function  $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$  for all  $x \in X$ . is called the degree of uncertainty of  $x \in A$ . The class of IFSs on a universe  $X$  is denoted by  $IFS(X)$ .

**Definition 0.5** ([2, 3]) If  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X \}$  be any two IFS of a set  $X$  then

$$\begin{aligned} A \subseteq B & \text{ iff for all } x \in X, \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \\ A = B & \text{ iff for all } x \in X, \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x) \\ A \cap B & = \{ \langle x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x) \rangle \mid x \in X \} \text{ where} \\ & (\mu_A \cap \mu_B)(x) = \min\{\mu_A(x), \mu_B(x)\} \text{ and } (\nu_A \cup \nu_B)(x) = \max\{\nu_A(x), \nu_B(x)\} \\ A \cup B & = \{ \langle x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x) \rangle \mid x \in X \} \text{ where} \\ & (\mu_A \cup \mu_B)(x) = \max\{\mu_A(x), \mu_B(x)\} \text{ and } (\nu_A \cap \nu_B)(x) = \min\{\nu_A(x), \nu_B(x)\} \end{aligned}$$

**Definition 0.6** ([4]) For any IFS  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$  of  $X$  and  $\alpha \in [0, 1]$ , the operator  $\square : IFS(X) \rightarrow IFS(X), \diamond : IFS(X) \rightarrow IFS(X), D_\alpha : IFS(X) \rightarrow IFS(X)$  are defined as

- (i)  $\square(A) = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X \}$  is called necessity operator
- (ii)  $\diamond(A) = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle \mid x \in X \}$  is called possibility operator
- (iii)  $D_\alpha(A) = \{ \langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1 - \alpha)\pi_A(x) \rangle \mid x \in X \}$  is called  $\alpha$ - Model operator. Clearly  $\square(A) \subseteq A \subseteq \diamond(A)$  and the equality hold, when  $A$  is a fuzzy set also  $D_0(A) = \square(A)$  and  $D_1(A) = \diamond(A)$ . Therefore the  $\alpha$ - Model operator  $D_\alpha(A)$  is an extension of necessity operator  $\square(A)$  and possibility operator  $\diamond(A)$ .

**Definition 0.7** ([4]) For any IFS  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$  of  $X$  and for any  $\alpha, \beta \in [0, 1]$  such that  $\alpha + \beta \leq 1$ , the  $(\alpha, \beta)$ - model operator  $F_{\alpha, \beta} : IFS(X) \rightarrow IFS(X)$  is defined as  $F_{\alpha, \beta}(A) = \{ \langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x) \rangle \mid x \in X \}$ , where  $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$  for all  $x \in X$ . Therefore we can write

$$F_{\alpha, \beta}(A) \text{ as } F_{\alpha, \beta}(A)(x) = (\mu_{F_{\alpha, \beta}(A)}(x), \nu_{F_{\alpha, \beta}(A)}(x))$$

where  $\mu_{F_{\alpha, \beta}(A)}(x) = \mu_A(x) + \alpha\pi_A(x)$  and  $\nu_{F_{\alpha, \beta}(A)}(x) = \nu_A(x) + \beta\pi_A(x)$ .  
Clearly,  $F_{0, 1}(A) = \square(A), F_{1, 0}(A) = \diamond(A)$  and  $F_{\alpha, 1-\alpha}(A) = D_\alpha(A)$

**Definition 0.8** ([9]) An intuitionistic fuzzy set  $A = (\mu_A, \nu_A)$  of a BG-algebra  $X$  is said to be an intuitionistic fuzzy subalgebra of  $X$  if

- (i)  $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$
- (ii)  $\nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\} \quad \forall x, y \in X$ .

**Definition 0.9** ([7]) An IFS  $A$  of a BG-algebra  $X$  is said to be an IF normal subalgebra of  $X$  if

- (i)  $\mu_A((x * a) * (y * b)) \geq \min\{\mu_A(x * y), \mu_A(a * b)\}$ ,
- (ii)  $\nu_A((x * a) * (y * b)) \leq \max\{\nu_A(x * y), \nu_A(a * b)\}$ ,  $\forall x, y \in X$ .

**Definition 0.10** ([11]) Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy set of BG-algebra  $X$ . Let  $t \in [0, 1]$ . then the intuitionistic fuzzy set  $A^t$  of  $X$  is called  $t$ -intuitionistic fuzzy subset ( $t$ -IFS) of  $X$  w.r.t  $A$  and is defined by  $A^t = \{ \langle x, \mu_{A^t}(x), \nu_{A^t}(x) \rangle \mid x \in X \} = \langle \mu_{A^t}, \nu_{A^t} \rangle$  where  $\mu_{A^t}(x) = \min\{\mu_A(x), t\}$  and  $\nu_{A^t} = \max\{\nu_A(x), 1 - t\} \forall x \in X$

**Remark 0.11** ([11]) Let  $A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle$  and  $B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle$  be two  $t$ -intuitionistic fuzzy subsets of BG-algebra  $X$ , then

$$(A \cap B)^t = A^t \cap B^t$$

**Remark 0.12** ([11]) Let  $f : X \rightarrow Y$  be a mapping. Let  $A$  and  $B$  are two IFS of  $X$  and  $Y$  respectively, then

$$(i) f^{-1}(B^t) = (f^{-1}(B))^t \quad (ii) f(A^t) = (f(A))^t \quad \forall t \in [0, 1]$$

**Definition 0.13** Let  $A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle$  and  $B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle$  be two  $t$ -intuitionistic fuzzy subsets of BG-algebra  $X$ . Then their cartesian product  $A^t \times B^t = \langle \mu_{A^t \times B^t}, \nu_{A^t \times B^t} \rangle$  is defined by

$$\begin{aligned} \mu_{A^t \times B^t}(x, y) &= \min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \\ \nu_{A^t \times B^t}(x, y) &= \max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \quad \forall x, y \in X. \end{aligned}$$

### t-Intuitionistic Fuzzy Subalgebra BG-algebra

Now onwards, let  $X$  denote a BG-algebra unless otherwise stated.

**Definition 0.14** Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy set of BG-algebra  $X$ . Let  $t \in [0, 1]$  then  $A$  is called  $t$ -intuitionistic fuzzy subalgebra ( $t$ -IFSA) of  $X$  if  $A^t$  is IFSA of  $X$  i.e. if  $A^t$  satisfies following conditions:

$$\begin{aligned} \mu_{A^t}(x * y) &\geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \\ \nu_{A^t}(x * y) &\leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \end{aligned}$$

**Theorem 0.15** If  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy subalgebra BG-algebra  $X$ , then  $A$  is also  $t$ -intuitionistic fuzzy subalgebra of  $X$ .

*Proof.* Since  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy subalgebra BG-algebra  $X$ , therefore

$$\begin{aligned} \mu_A(x * y) &\geq \min\{\mu_A(x), \mu_A(y)\} \\ \nu_A(x * y) &\leq \max\{\nu_A(x), \nu_A(y)\}, \quad \forall x, y \in X. \end{aligned}$$

$$\begin{aligned} \text{Now, } \mu_{A^t}(x * y) &= \min\{\mu_A(x * y), t\} \\ &\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, t\} \\ &= \min\{\min\{\mu_A(x), t\}, \min\{\mu_A(y), t\}\} \\ &= \min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \\ \Rightarrow \mu_{A^t}(x * y) &\geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \end{aligned}$$

Similarly we can show

$$\nu_{A^t}(x * y) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$$

Hence  $A$  is also  $t$ -intuitionistic fuzzy subalgebra BG-algebra  $X$ .

**Remark 0.16** *The converse of above Theorem is not true.*

*Example 1.* Consider a BG-algebra  $X = \{0, 1, 2\}$  with the following cayley table:

Table 1: Example of intuitionistic fuzzy BG-subalgebra.

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

The intuitionistic fuzzy subset  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$  given by  $\mu_A(0) = 0.4$ ,  $\mu_A(1) = 0.5$ ,  $\mu_A(2) = 0.3$  and  $\nu_A(0) = 0.5$ ,  $\nu_A(1) = 0.4$ ,  $\nu_A(2) = 0.6$ . Since  $\mu_A(0) = 0.4 \not\geq \min\{\mu_A(1), \mu_A(2)\}$ . Therefore  $A$  is not an intuitionistic fuzzy BG-subalgebra of  $X$ . Take  $t = 0.2$ . Then  $\mu_A(x) > t$  for all  $x \in X$  and also  $\nu_A(x) < 1 - t$  for all  $x \in X$ . Therefore  $\mu_{A^t}(x * y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$  and  $\nu_{A^t}(x * y) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$  for all  $x \in X$ . hold. Hence  $A$  is t-intuitionistic fuzzy subalgebra of  $X$ .

**Theorem 0.17** *If  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy set of BG-algebra  $X$  and let  $t < \min\{p, 1 - q\}$ , where  $p = \min\{\mu_A(x) \mid x \in X\}$  and  $q = \max\{\nu_A(x) \mid x \in X\}$  then  $A$  is also t-intuitionistic fuzzy subalgebra BG-algebra  $X$ .*

*Proof.* Since  $t < \min\{p, 1 - q\}$

$$\begin{aligned}
 t &< \min\{p, 1 - q\} \\
 \Rightarrow p &> t \quad \text{and} \quad 1 - q > t \\
 \Rightarrow p &> t \quad \text{and} \quad q < 1 - t \\
 \Rightarrow \min\{\mu_A(x) \mid x \in X\} &> t \quad \text{and} \quad \max\{\nu_A(x) \mid x \in X\} < 1 - t \\
 \Rightarrow \mu_A(x) &> t, \forall x \in X \quad \text{and} \quad \nu_A(x) < 1 - t, \forall x \in X
 \end{aligned}$$

Therefore  $\mu_{A^t}(x * y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$  and  $\nu_{A^t}(x * y) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$  for all  $x \in X$  hold. Hence  $A$  is t-intuitionistic fuzzy subalgebra of  $X$ .

**Theorem 0.18** *Any IF set of BG-algebra  $X$  can be realised as t-intuitionistic fuzzy subalgebra  $X$ .*

*Proof.* It follows from Theorem 0.17 and Theorem 0.15.

**Theorem 0.19** *The intersection of two t-intuitionistic fuzzy subalgebra BG-algebra  $X$  is also a t-intuitionistic fuzzy subalgebra of  $X$ .*

*Proof.* Let  $x, y \in X$ . Then

$$\begin{aligned}
\mu_{(A \cap B)^t}(x * y) &= \min\{\mu_{(A \cap B)}(x * y), t\} \\
&\geq \min\{\min\{\mu_A(x * y), \mu_A(x * y)\}, t\} \\
&= \min\{\min(\mu_A(x * y), t), \min(\mu_B(x * y), t)\} \\
&= \min\{\mu_{A^t}(x * y), \mu_{B^t}(x * y)\} \\
&\geq \min\{\min\{\mu_{A^t}(x), \mu_{A^t}(y)\}, \min\{\mu_{B^t}(x), \mu_{B^t}(y)\}\} \\
&= \min\{\min\{\mu_{A^t}(x), \mu_{B^t}(x)\}, \min\{\mu_{A^t}(y), \mu_{B^t}(y)\}\} \\
&= \min\{\mu_{(A \cap B)^t}(x), \mu_{(A \cap B)^t}(y)\} \\
\Rightarrow \mu_{(A \cap B)^t}(x * y) &\geq \min\{\mu_{(A \cap B)^t}(x), \mu_{(A \cap B)^t}(y)\}
\end{aligned}$$

Similarly we can show that

$$\nu_{(A \cap B)^t}(x * y) \leq \max\{\nu_{(A \cap B)^t}(x), \nu_{(A \cap B)^t}(y)\}$$

**Theorem 0.20** *The intersection of any number of  $t$ -intuitionistic fuzzy subalgebra BG-algebra  $X$  is also a  $t$ -intuitionistic fuzzy subalgebra of  $X$ .*

**Theorem 0.21** *For every  $t$ -intuitionistic fuzzy subalgebra  $A^t$  of  $X$ , the following properties hold*

- (i)  $\mu_{A^t}(0) \geq \mu_{A^t}(x)$
- (ii)  $\nu_{A^t}(0) \leq \nu_{A^t}(x), \forall x \in X$ .

*Proof.* We have  $\mu_{A^t}(0) = \mu_{A^t}(x * x) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(x)\} = \mu_{A^t}(x)$   
and  $\nu_{A^t}(0) = \nu_{A^t}(x * x) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(x)\} = \nu_{A^t}(x)$

**Theorem 0.22** *If  $A$  be IF subalgebra of BG-algebra  $X$ , then  $\square A, \diamond A$  and  $F_{\alpha, \beta}(A)$  are also  $t$ -intuitionistic fuzzy subalgebra of  $X$ .*

*Proof.* Here  $A$  be IF subalgebra of BG-algebra  $X$ , By Theorem 0.15  $A$  is also  $t$ -intuitionistic fuzzy subalgebra of  $X$ .

$$\mu_{A^t}(x * y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \quad (1)$$

$$\nu_{A^t}(x * y) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \quad \forall x, y \in X. \quad (2)$$

Now  $\square A^t = \{\langle x, \mu_{A^t}(x), 1 - \mu_{A^t}(x) \mid x \in X \rangle = \{\langle x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) \mid x \in X \rangle\}$   
 $\diamond A^t = \{\langle x, 1 - \nu_{A^t}(x), \mu_{A^t}(x) \mid x \in X \rangle = \{\langle x, \overline{\nu_{A^t}}(x), \mu_{A^t}(x) \mid x \in X \rangle\}$

Now

$$\begin{aligned}
\overline{\mu_{A^t}}(x * y) &= 1 - \mu_{A^t}(x * y) \\
&\leq 1 - \min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \quad \text{By(1)} \\
&= \max\{1 - \mu_{A^t}(x), 1 - \mu_{A^t}(y)\} \\
&= \max\{\overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y)\}
\end{aligned}$$

$$\Rightarrow \overline{\mu_{A^t}}(x * y) \leq \max\{\overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y)\} \quad (3)$$

Hence by Eq<sup>n</sup> (1) and (3)  $\Rightarrow \square A^t = \{\langle x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) \mid x \in X \rangle$  is  $t$ -intuitionistic fuzzy subalgebra of  $X$ .

Similarly we can show that

$\diamond A^t = \{ \langle x, \overline{\nu_{A^t}}(x), \mu_{A^t}(x) \mid x \in X \}$  is t-intuitionistic fuzzy subalgebra of X.

Again, we have  $F_{\alpha,\beta}(A) = \langle \mu_{F_{\alpha,\beta}(A)}, \nu_{F_{\alpha,\beta}(A)} \rangle$  let  $x, y \in X$ , then  $F_{\alpha,\beta}(x * y) = (\mu_{F_{\alpha,\beta}(A)}(x * y), \nu_{F_{\alpha,\beta}(A)}(x * y))$  where  $\mu_{F_{\alpha,\beta}(A)}(x * y) = \mu_A(x * y) + \alpha\pi_A(x * y)$  and  $\nu_{F_{\alpha,\beta}(A)}(x * y) = \nu_A(x * y) + \beta\pi_A(x * y)$

$$\begin{aligned} & \mu_{F_{\alpha,\beta}A^t}(x * y) \\ &= \mu_{A^t}(x * y) + \alpha\pi_{A^t}(x * y) \\ &= \mu_{A^t}(x * y) + \alpha(1 - \mu_{A^t}(x * y) - \nu_{A^t}(x * y)) \\ &\geq \alpha + (1 - \alpha)\mu_{A^t}(x * y) - \alpha\nu_{A^t}(x * y) \\ &\geq \alpha + (1 - \alpha)\min(\mu_{A^t}(x), \mu_{A^t}(y)) - \alpha\max(\nu_{A^t}(x), \nu_{A^t}(y)) \quad \text{By(1)} \\ &\geq \alpha\{1 - \max(\nu_{A^t}(x), \nu_{A^t}(y))\} + (1 - \alpha)\min(\mu_{A^t}(x), \mu_{A^t}(y)) \\ &\geq \alpha\min(1 - \nu_{A^t}(x), 1 - \nu_{A^t}(y))\} + (1 - \alpha)\min(\mu_{A^t}(x), \mu_{A^t}(y)) \\ &\geq \min\{\alpha(1 - \nu_{A^t}(x)) + (1 - \alpha)\mu_{A^t}(x), \alpha(1 - \nu_{A^t}(y)) + (1 - \alpha)\mu_{A^t}(y)\} \\ &\geq \min\{\mu_{A^t}(x) + \alpha(1 - \mu_{A^t}(x) - \nu_{A^t}(x)), \mu_{A^t}(y) + \alpha(1 - \mu_{A^t}(y) - \nu_{A^t}(y))\} \\ &\geq \min\{\mu_{F_{\alpha,\beta}A^t}(x), \mu_{F_{\alpha,\beta}A^t}(y)\} \end{aligned}$$

$$\therefore \mu_{F_{\alpha,\beta}A^t}(x * y) \geq \min\{\mu_{F_{\alpha,\beta}A^t}(x), \mu_{F_{\alpha,\beta}A^t}(y)\}$$

Similarly we can prove that

$$\nu_{F_{\alpha,\beta}A^t}(x * y) \leq \max\{\nu_{F_{\alpha,\beta}A^t}(x), \nu_{F_{\alpha,\beta}A^t}(y)\}$$

Hence  $F_{\alpha,\beta}(A)$  is t-intuitionistic fuzzy subalgebra of X.

**Theorem 0.23** Cartesian product of two t-intuitionistic fuzzy subalgebra of X is again a t-intuitionistic fuzzy subalgebra of  $X \times X$ .

*Proof.* Let  $A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle$  and  $B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle$  be two t-intuitionistic fuzzy subalgebra of BG-algebra X

Then their cartesian product  $A^t \times B^t = \langle \mu_{A^t \times B^t}, \nu_{A^t \times B^t} \rangle$ , where

$$\begin{aligned} \mu_{A^t \times B^t}(x, y) &= \min\{\mu_{A^t}(x), \mu_{B^t}(y)\} \\ \nu_{A^t \times B^t}(x, y) &= \max\{\nu_{A^t}(x), \nu_{B^t}(y)\} \quad \forall x, y \in X. \end{aligned}$$

Also

$$\mu_{A^t}(x * y) \geq \min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \tag{4}$$

$$\nu_{A^t}(x * y) \leq \max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \quad \forall x, y \in X. \tag{5}$$

$$\begin{aligned} \mu_{A^t \times B^t}((x_1, y_1) * (x_2, y_2)) &= \mu_{A^t \times B^t}(x_1 * x_2, y_1 * y_2) \\ &= \min\{\mu_{A^t}(x_1 * x_2), \mu_{B^t}(y_1 * y_2)\} \\ &\geq \min\{\min\{\mu_{A^t}(x_1), \mu_{A^t}(x_2)\}, \min\{\mu_{B^t}(y_1), \mu_{B^t}(y_2)\}\} \\ &= \min\{\min\{\mu_{A^t}(x_1), \mu_{B^t}(y_1)\}, \min\{\mu_{A^t}(x_2), \mu_{B^t}(y_2)\}\} \\ &= \min\{\mu_{A^t \times B^t}((x_1, y_1), \mu_{A^t \times B^t}((x_2, y_2))\} \\ \Rightarrow \mu_{A^t \times B^t}((x_1, y_1) * (x_2, y_2)) &\geq \min\{\mu_{A^t \times B^t}((x_1, y_1), \mu_{A^t \times B^t}((x_2, y_2))\} \end{aligned}$$

Similarly we can show

$$\nu_{A^t \times B^t}((x_1, y_1) * (x_2, y_2)) \leq \max\{\nu_{A^t \times B^t}((x_1, y_1), \nu_{A^t \times B^t}((x_2, y_2))\}$$

**Corollary 0.24** If  $A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle$  and  $B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle$  be two  $t$ -intuitionistic fuzzy subalgebra of BG-algebra  $X$ . Then  $\square(A^t \times B^t)$ ,  $\diamond(A^t \times B^t)$ ,  $F_{\alpha, \beta}(A^t \times B^t)$  are also  $t$ -intuitionistic fuzzy subalgebra of  $X \times X$ .

**Theorem 0.25** If  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy normal subalgebra BG-algebra  $X$ , then  $A$  is also  $t$ -intuitionistic fuzzy normal subalgebra of  $X$ .

*Proof.* Since  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy normal subalgebra BG-algebra  $X$ , therefore

$$\begin{aligned} (i) \mu_A((x * a) * (y * b)) &\geq \min\{\mu_A(x * y), \mu_A(a * b)\} \\ (ii) \nu_A((x * a) * (y * b)) &\leq \max\{\nu_A(x * y), \nu_A(a * b)\}, \forall x, y \in X. \end{aligned}$$

$$\begin{aligned} \text{Now, } \mu_{A^t}((x * a) * (y * b)) &= \min\{\mu_A((x * a) * (y * b)), t\} \\ &\geq \min\{\min\{\mu_A(x * y), \mu_A(a * b)\}, t\} \\ &= \min\{\min(\mu_A(x * y), t), \min(\mu_A(a * b), t)\} \\ &= \min\{\mu_{A^t}(x * y), \mu_{A^t}(a * b)\} \\ \Rightarrow \mu_{A^t}((x * a) * (y * b)) &\geq \min\{\mu_{A^t}(x * y), \mu_{A^t}(a * b)\} \end{aligned}$$

Similarly we can show that

$$\nu_{A^t}((x * a) * (y * b)) \leq \max\{\nu_{A^t}(x * y), \nu_{A^t}(a * b)\}$$

Hence  $A$  is also  $t$ -intuitionistic fuzzy normal subalgebra BG-algebra  $X$ .

**Remark 0.26** The converse of above Theorem is not true.

**Theorem 0.27** If  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy set of BG-algebra  $X$  and let  $t < \min\{p, 1 - q\}$ , where  $p = \min\{\mu_A(x) | x \in X\}$  and  $q = \max\{\nu_A(x) | x \in X\}$  then  $A$  is also  $t$ -intuitionistic fuzzy normal subalgebra BG-algebra  $X$ .

*Proof.* Same as Theorem 0.17.

**Theorem 0.28** The intersection of two  $t$ -intuitionistic fuzzy normal subalgebra BG-algebra  $X$  is also a  $t$ -intuitionistic fuzzy normal subalgebra of  $X$ .

*Proof.* Same as Theorem 0.19.

**Theorem 0.29** If  $A$  be IF normal subalgebra of BG-algebra  $X$ , then  $\square A$ ,  $\diamond A$  and  $F_{\alpha, \beta}(A)$  are also  $t$ -intuitionistic fuzzy normal subalgebra of  $X$ .

*Proof.* Same as Theorem 0.22.

**Theorem 0.30** Cartesian product of two  $t$ -intuitionistic fuzzy normal subalgebra of  $X$  is again a  $t$ -intuitionistic fuzzy normal subalgebra of  $X \times X$ .

*Proof.* Same as Theorem 0.23.

**Corollary 0.31** If  $A^t = \langle \mu_{A^t}, \nu_{A^t} \rangle$  and  $B^t = \langle \mu_{B^t}, \nu_{B^t} \rangle$  be two  $t$ -intuitionistic fuzzy normal subalgebra of BG-algebra  $X$ . Then  $\square(A^t \times B^t)$ ,  $\diamond(A^t \times B^t)$ ,  $F_{\alpha, \beta}(A^t \times B^t)$  are also  $t$ -intuitionistic fuzzy normal subalgebra of  $X \times X$ .

*Proof.* Same as Corollary 0.24.

**Homomorphism of t-intuitionistic fuzzy subalgebra BG-algebra**

**Definition 0.32** Let  $X$  and  $Y$  be two BG-algebras, then a mapping  $f : X \rightarrow Y$  is said to be homomorphism if  $f(x * y) = f(x) * f(y), \forall x, y \in X$ .

**Theorem 0.33** Let  $f : X \rightarrow Y$  be a homomorphism of BG-algebras, If  $A$  be a t-intuitionistic fuzzy subalgebra of  $Y$ , then  $f^{-1}(A)$  is t-intuitionistic fuzzy subalgebra  $X$ .

*Proof.*  $A$  be a t-intuitionistic fuzzy subalgebra of  $Y$ . Let  $x, y \in X$  be any elements, then  $f^{-1}(A^t)(x * y) = (\mu_{f^{-1}(A^t)}(x * y), \nu_{f^{-1}(A^t)}(x * y))$

$$\begin{aligned} &\text{Now, } \mu_{f^{-1}(A^t)}(x * y) \\ &= \mu_{A^t} f(x * y) \\ &= \mu_{A^t} [f(x) * f(y)] \\ &\geq \min\{\mu_{A^t}(f(x)), \mu_{A^t}(f(y))\} \quad [\text{Since } A \text{ is t-IF subalgebra of } Y] \\ &= \min\{\mu_{f^{-1}A^t}(x), \mu_{f^{-1}A^t}(y)\} \end{aligned}$$

Therefore  $\mu_{f^{-1}A^t}(x * y) \geq \min\{\mu_{f^{-1}A^t}(x), \mu_{f^{-1}A^t}(y)\}$

Similarly we can show that

$$\nu_{f^{-1}(A^t)}(x * y) \leq \max\{\nu_{f^{-1}A^t}(x), \nu_{f^{-1}A^t}(y)\}$$

Hence,  $f^{-1}(A^t) = (f^{-1}(A))^t$  is t-intuitionistic fuzzy subalgebra  $X$ .

**Theorem 0.34** Let  $f : X \rightarrow Y$  be a homomorphism of BG-algebras, If  $A$  be a t-intuitionistic fuzzy normal subalgebra of  $Y$ , then  $f^{-1}(A)$  is t-intuitionistic fuzzy normal subalgebra  $X$ .

**Theorem 0.35** Let  $f : X \rightarrow Y$  be a onto homomorphism of BG-algebras, If  $A$  be t-intuitionistic fuzzy subalgebra  $X$ . Then  $f(A)$  is t-intuitionistic fuzzy subalgebra of  $Y$ .

*Proof.* Let  $y_1, y_2 \in Y$  Since  $f$  is onto, therefore there exists  $x_1, x_2 \in X$  such that  $f(x_1) = y_1, f(x_2) = y_2$ ,

$$f(A)(y_1 * y_2) = (\mu_{f(A)}(y_1 * y_2), \nu_{f(A)}(y_1 * y_2)), \text{ Now}$$

$$\mu_{f(A)}(y_1 * y_2) = \mu_A(t) \text{ where } f(t) = y_1 * y_2 = f(x_1) * f(x_2) = f(x_1 * x_2)$$

$$\begin{aligned} &\mu_{f(A^t)}(y_1 * y_2) \\ &= \mu_{(f(A))^t}(y_1 * y_2) \\ &= \min\{\mu_{f(A)}(y_1 * y_2), t\} \\ &= \min\{\mu_{f(A)}(f(x_1) * f(x_2)), t\} \\ &= \min\{\mu_{f(A)}f(x_1 * x_2), t\} \\ &= \min\{\mu_A(x_1 * x_2), t\} \\ &= \mu_{A^t}(x_1 * x_2) \\ &\geq \min\{\mu_{A^t}(x_1), \mu_{A^t}(x_2)\}, \text{ for all } x_1, x_2 \in X \text{ such that } f(x_1) = y_1 \text{ and } f(x_2) = y_2 \\ &= \min\{\bigvee\{\mu_{A^t}(x_1) | f(x_1) = y_1\}, \bigvee\{\mu_{A^t}(x_2) | f(x_2) = y_2\}\} \\ &= \min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\} \end{aligned}$$

Therefore  $\mu_{f(A^t)}(y_1 * y_2) \geq \min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$

Similarly we can show that

$$\nu_{f(A^t)}(y_1 * y_2) \leq \max\{\nu_{f(A^t)}(y_1), \nu_{f(A^t)}(y_2)\}$$

Hence  $f(A)$  is t-intuitionistic fuzzy subalgebra of  $Y$ .

**Theorem 0.36** *Let  $f : X \longrightarrow Y$  be a onto homomorphism of BG-algebras, If  $A$  be  $t$ -intuitionistic fuzzy normal subalgebra  $X$ , then  $f(A)$  is  $t$ -intuitionistic fuzzy normal subalgebra of  $Y$ .*

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