t- Intuitionistic Fuzzy Subalgebra of $BG$-Algebras

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Abstract. The aim of this paper is to introduced the notion of t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra of $BG$-algebras. We state and prove some theorems in t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra in $BG$-algebras. The homomorph image and inverse image are investigated in both t-intuitionistic fuzzy subalgebra and normal subalgebras.

Introduction

In 1966, Imai and Iseki [6] introduced the two classes of abstract algebras, viz., $BCK$-algebras and $BCI$-algebras. It is known that the class of $BCK$-algebra is a proper subclass of the class of $BCI$-algebras. Neggers and Kim [8] introduced a new concept, called $B$-algebras, which are related to several classes of algebras such as $BCI/BCK$-algebras. Kim and Kim [7] introduced the notion of $BG$-algebra which is a generalization of $B$-algebra. The concept of intuitionistic fuzzy subset (IFS) was introduced by Atanassov [5] in 1983, which is a generalization of the notion of fuzzy sets. The concept of fuzzy subalgebras of $BG$-algebras was introduced by Ahn and Lee in [1]. The study of intuitionistic fuzzification of subalgebras and ideals of $BG$-algebras is done by Senapati et. al in [9]. The idea of t-intuitionistic fuzzy sets in fuzzy subgroups and fuzzy subrings is introduced by Sharma in [10, 11]. Here in this paper, we introduced the notion of t-intuitionistic fuzzy sets in fuzzy subalgebra and fuzzy normal subalgebras of $BG$-algebras and study their properties.

Preliminaries

Definition 0.1 ([1]) A $BG$-algebra is a non-empty set $X$ with a constant $'0'$ and a binary operation $'*'$ satisfying the following axioms:

(i) $x * x = 0$,

(ii) $x * 0 = x$,

(iii) $(x * y) * (0 * y) = x, \forall x, y \in X$.

For brevity, we also call $X$ a $BG$-algebra. We can define a partial ordering $'\leq'$ on $X$ by $x \leq y$ iff $x * y = 0$.

Definition 0.2 ([1]) A non-empty subset $S$ of a $BG$-algebra $X$ is called a subalgebra of $X$ if $x * y \in S$, for all $x, y \in S$. 
Definition 0.3 Let $X$ and $Y$ be two non-empty sets and $f : X \rightarrow Y$ be a mapping. Let $A$ and $B$ be IFS's of $X$ and $Y$ respectively. Then the image of $A$ under the map $f$ is denoted by $f(A)$ and is defined by $f(A)(y) = (\mu_f(A)y, \nu_f(A)y)$, where

$$
\nu_f(A)(y) = \begin{cases}
\bigwedge \{\nu_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\
1 & \text{otherwise}
\end{cases}
$$

also preimage of $B$ under $f$ is denoted by $f^{-1}(B)$ and is defined as $f^{-1}(B)(x) = (\mu_{f^{-1}}(B), \nu_{f^{-1}}(B)) = (\mu_B(f(x)), \nu_B(f(x)))$ for all $x \in X$

Remark Note that $f_\triangledown$ is bijective.

Definition 0.6 For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X \}$ of $X$ and $\alpha \in [0, 1]$, the operator $\square : IFS(X) \rightarrow IFS(X)$, $\triangledown : IFS(X) \rightarrow IFS(X)$, $D_{\alpha} : IFS(X) \rightarrow IFS(X)$ are defined as

(i) $\square(A) = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X \}$ is called necessity operator

(ii) $\triangledown(A) = \{< x, 1 - \nu_A(x), \nu_A(x) > | x \in X \}$ is called possibility operator

(iii) $D_{\alpha}(A) = \{< x, \alpha \pi_A(x), \nu_A(x) + (1 - \alpha) \pi_A(x) > | x \in X \}$ is called $\alpha$-Model operator.

Clearly $\square(A) \subseteq A \subseteq \triangledown(A)$ and the equality hold, when $A$ is a fuzzy set also $D_{\alpha}(A) = \square(A)$ and $D_{1}(A) = \triangledown(A)$. Therefore the $\alpha$-Model operator $D_{\alpha}(A)$ is an extension of necessity operator $\square(A)$ and possibility operator $\triangledown(A)$.

Definition 0.7 For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X \}$ of $X$ and for any $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$, the $(\alpha, \beta)$-model operator $F_{\alpha, \beta} : IFS(X) \rightarrow IFS(X)$ is defined as

$$
F_{\alpha, \beta}(A)(x) = (\mu_{F_{\alpha, \beta}}(A)(x), \nu_{F_{\alpha, \beta}}(A)(x))
$$

where $\mu_{F_{\alpha, \beta}}(x) = \mu_A(x) + \alpha \pi_A(x)$ and $\nu_{F_{\alpha, \beta}}(A)(x) = \nu_A(x) + \beta \pi_A(x)$.

Clearly, $F_{0,1}(A) = \square(A)$, $F_{1,0}(A) = \triangledown(A)$ and $F_{\alpha, 1-\alpha}(A) = D_{\alpha}(A)$

Definition 0.8 (9) An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of a BG-algebra of $X$ is said to be an intuitionistic fuzzy subalgebra of $X$ if

(i)$\mu_A(x \ast y) \geq \min\{\mu_A(x), \mu_A(y)\}$

(ii)$\nu_A(x \ast y) \leq \max\{\nu_A(x), \nu_A(y)\}$

$\forall x, y \in X.$
Definition 0.9 ([7]) An IFS $A$ of a BG-algebra $X$ is said to be an IF normal subalgebra of $X$ if:

(i) $\mu_A((x * a) * (y * b)) \geq \min\{\mu_A(x * y), \mu_A(a * b)\}$,

(ii) $\nu_A((x * a) * (y * b)) \leq \max\{\nu_A(x * y), \nu_A(a * b)\}$, $\forall x, y \in X$.

Definition 0.10 ([11]) Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of BG-algebra $X$. Let $t \in [0, 1]$, then the intuitionistic fuzzy set $A^t$ of $X$ is called $t$-intuitionistic fuzzy subset (t-IFS) of $X$ w.r.t $A$ and is defined by $A^t = \{<x, \mu_A^t(x), \nu_A^t(x)> | x \in X\} =<\mu_A^t, \nu_A^t>$ where $\mu_A^t(x) = \min\{\mu_A(x), t\}$ and $\nu_A^t = \max\{\nu_A(x), 1 - t\}$ $\forall x \in X$.

Remark 0.11 ([11]) Let $A^t =<\mu_A^t, \nu_A^t>$ and $B^t =<\mu_B^t, \nu_B^t>$ be two $t$-intuitionistic fuzzy subsets of BG-algebra $X$, then

$(A \cap B)^t = A^t \cap B^t$

Remark 0.12 ([11]) Let $f : X \to Y$ be a mapping. Let $A$ and $B$ are two IFS of $X$ and $Y$ respectively, then

(i) $f^{-1}(B^t) = (f^{-1}(B))^t$ (ii) $f(A^t) = (f(A))^t \ \forall t \in [0, 1]$.

Definition 0.13 Let $A^t =<\mu_A^t, \nu_A^t>$ and $B^t =<\mu_B^t, \nu_B^t>$ be two $t$-intuitionistic fuzzy subsets of BG-algebra $X$ Then their cartesian product $A^t \times B^t =<\mu_{A^t \times B^t}, \nu_{A^t \times B^t}>$ is defined by

$\mu_{A^t \times B^t}(x, y) = \min\{\mu_A^t(x), \mu_B^t(y)\}$

$\nu_{A^t \times B^t}(x, y) = \max\{\nu_A^t(x), \nu_B^t(y)\}$ $\forall x, y \in X$.

$t$-Intuitionistic Fuzzy Subalgebra BG-algebra

Now onwards, let $X$ denote a BG-algebra unless otherwise stated.

Definition 0.14 Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of BG-algebra $X$. Let $t \in [0, 1]$ then $A$ is called $t$-intuitionistic fuzzy subalgebra (t-IFSA) of $X$ if $A^t$ is IFSA of $X$ i.e. if $A^t$ satisfies following conditions:

$\mu_A^t(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$

$\nu_A^t(x * y) \leq \max\{\nu_A(x), \nu_A(y)\}$

Theorem 0.15 If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subalgebra BG-algebra $X$, then $A$ is also $t$-intuitionistic fuzzy subalgebra of $X$.

Proof. Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subalgebra BG-algebra $X$, therefore

$\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$

$\nu_A(x * y) \leq \max\{\nu_A(x), \nu_A(y)\}$, $\forall x, y \in X$.

Now, $\mu_A^t(x * y) = \min\{\mu_A(x * y), t\}$

$\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, t\}$

$= \min\{\min(\mu_A(x), t), \min(\mu_A(y), t)\}$

$= \min\{\mu_A^t(x), \mu_A^t(y)\}$

$\Rightarrow \mu_A^t(x * y) \geq \min\{\mu_A^t(x), \mu_A^t(y)\}$

Similarly we can show

$\nu_A^t(x * y) \leq \max\{\nu_A^t(x), \nu_A^t(y)\}$

Hence $A$ is also $t$-intuitionistic fuzzy subalgebra BG-algebra $X$. 

Remark 0.16  

The converse of above Theorem is not true.

Example 1. Consider a BG-algebra $X = \{0, 1, 2\}$ with the following Cayley table:

Table 1: Example of intuitionistic fuzzy BG-subalgebra.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The intuitionistic fuzzy subset $A = \{x, \mu_A(x), \nu_A(x) \mid x \in X\}$ given by $\mu_A(0) = 0.4, \mu_A(1) = 0.5, \mu_A(2) = 0.3$ and $\nu_A(0) = 0.5, \nu_A(1) = 0.4, \nu_A(2) = 0.6$. Since $\mu_A(0) = 0.4 \geq \min\{\mu_A(1), \mu_A(1)\}$. Therefore $A$ is not an intuitionistic fuzzy BG-subalgebra of $X$.

Take $t = 0.2$. Then $\mu_A(x) > t$ for all $x \in X$ and also $\nu_A(x) < 1 - t$ for all $x \in X$.

Therefore $\mu_A(x \ast y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x \ast y) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all $x \in X$.

Hence $A$ is $t$-intuitionistic fuzzy subalgebra of $X$.

Theorem 0.17  If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set of BG-algebra $X$ and let $t < \min\{p, 1 - q\}$, where $p = \min\{\mu_A(x) \mid x \in X\}$ and $q = \max\{\nu_A(x) \mid x \in X\}$ then $A$ is also $t$-intuitionistic fuzzy subalgebra BG-algebra $X$.

Proof. Since $t < \min\{p, 1 - q\}$

\begin{align*}
    t &< \min\{p, 1 - q\} \\
    \Rightarrow & \quad p > t \quad \text{and} \quad 1 - q > t \\
    \Rightarrow & \quad p > t \quad \text{and} \quad q < 1 - t \\
    \Rightarrow & \quad \min\{\mu_A(x) \mid x \in X\} > t \quad \text{and} \quad \max\{\nu_A(x) \mid x \in X\} < 1 - t \\
    \Rightarrow & \quad \mu_A(x) > t, \forall \ x \in X \quad \text{and} \quad \nu_A(x) < 1 - t, \forall \ x \in X
\end{align*}

Therefore $\mu_A(x \ast y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(x \ast y) \leq \max\{\nu_A(x), \nu_A(y)\}$ for all $x \in X$.

Hence $A$ is $t$-intuitionistic fuzzy subalgebra of $X$.

Theorem 0.18  Any IF set of BG-algebra $X$ can be realised as $t$-intuitionistic fuzzy subalgebra $X$.

Proof. It follows from Theorem 0.17 and Theorem 0.15.

Theorem 0.19  The intersection of two $t$-intuitionistic fuzzy subalgebra BG-algebra $X$ is also a $t$-intuitionistic fuzzy subalgebra of $X$.
Proof. Let \( x, y \in X \). Then
\[
\mu_{(A \cap B)^t}(x \ast y) = \min \{ \mu_{(A \cap B)^t}(x \ast y), t \} \\
\geq \min \{ \min \{ \mu_A(x \ast y), \mu_A(x \ast y) \}, t \} \\
= \min \{ \min \{ \mu_A(x \ast y), t \}, \min \{ \mu_B(x \ast y), t \} \} \\
= \min \{ \mu_A(x \ast y), \mu_B(x \ast y) \} \\
\geq \min \{ \mu_A(x \ast y), \mu_B(x \ast y) \} \\
= \min \{ \mu_A(x \ast y), \mu_B(x \ast y) \} \\
= \min \{ \mu_{(A \cap B)^t}(x), \mu_{(A \cap B)^t}(y) \}
\]

Similarly we can show that
\[
\nu_{(A \cap B)^t}(x \ast y) \leq \max \{ \nu_{(A \cap B)^t}(x), \nu_{(A \cap B)^t}(y) \}
\]

Theorem 0.20 The intersection of any number of t-intuitionistic fuzzy subalgebra BG-algebra \( X \) is also a t-intuitionistic fuzzy subalgebra of \( X \).

Theorem 0.21 For every t-intuitionistic fuzzy subalgebra \( A^t \) of \( X \), the following properties hold
(i) \( \mu_{A^t}(0) \geq \mu_{A^t}(x) \)
(ii) \( \nu_{A^t}(0) \leq \nu_{A^t}(x) \), \( \forall x \in X \).

Proof. We have \( \mu_{A^t}(0) = \mu_{A^t}(x \ast x) \geq \min \{ \mu_{A^t}(x), \mu_{A^t}(x) \} = \mu_{A^t}(x) \)
and \( \nu_{A^t}(0) = \nu_{A^t}(x \ast x) \leq \max \{ \nu_{A^t}(x), \nu_{A^t}(x) \} = \nu_{A^t}(x) \)

Theorem 0.22 If \( A \) be IF subalgebra of BG-algebra \( X \), then \( \square A, \vee A \) and \( F_{\alpha,\beta}(A) \) are also t-intuitionistic fuzzy subalgebra of \( X \).

Proof. Here \( A \) be IF subalgebra of BG-algebra \( X \). By Theorem 0.15 \( A \) is also t-intuitionistic fuzzy subalgebra of \( X \).

\[
\mu_{A^t}(x \ast y) \geq \min \{ \mu_{A^t}(x), \mu_{A^t}(y) \} \quad (1) \\
\nu_{A^t}(x \ast y) \leq \max \{ \nu_{A^t}(x), \nu_{A^t}(y) \} \quad (2)
\]

Now
\[
\square A^t = \{ < x, \mu_{A^t}(x), 1 - \mu_{A^t}(x) | x \in X \} = \{ < x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) | x \in X \}
\]
\[
\vee A^t = \{ < x, 1 - \nu_{A^t}(x), \mu_{A^t}(x) | x \in X \} = \{ < x, \overline{\nu_{A^t}}(x), \mu_{A^t}(x) | x \in X \}
\]

Now
\[
\overline{\mu_{A^t}}(x \ast y) = 1 - \mu_{A^t}(x \ast y) \\
\leq 1 - \min \{ \mu_{A^t}(x), \mu_{A^t}(y) \} \quad \text{By(1)}
\]
\[
= \max \{ 1 - \mu_{A^t}(x), 1 - \mu_{A^t}(y) \} \\
= \max \{ \overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y) \}
\]
\[
\Rightarrow \overline{\mu_{A^t}}(x \ast y) \leq \max \{ \overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y) \}
\]

Hence by Eqn (1) and (3) \( \square A^t = \{ < x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) | x \in X \} \) is t-intuitionistic fuzzy subalgebra of \( X \).

Similarly we can show that
\( A' = \{ x, \mu_{A'}(x) \mid x \in X \} \) is t-intuitionistic fuzzy subalgebra of X.
Again, we have \( F_{\alpha,\beta}(A) = (\mu_{F_{\alpha,\beta}}(A), \nu_{F_{\alpha,\beta}}(A)) \) let \( x, y \in X \), then \( F_{\alpha,\beta}(x * y) = (\mu_{F_{\alpha,\beta}}(A)(x * y), \nu_{F_{\alpha,\beta}}(A)(x * y)) \) where \( \mu_{F_{\alpha,\beta}}(A)(x * y) = \mu_A(x * y) + \alpha \pi_A(x * y) \) and \( \nu_{F_{\alpha,\beta}}(A)(x * y) = \nu_A(x * y) + \beta \pi_A(x * y) \)

\[
\begin{align*}
\mu_{F_{\alpha,\beta}}(x * y) &= \mu_{A'}(x * y) + \alpha \pi_{A'}(x * y) \\
&= \mu_{A'}(x * y) + \alpha(1 - \mu_{A'}(x * y) - \nu_{A'}(x * y)) \\
&\geq \alpha + (1 - \alpha) \pi_{A'}(x * y) - \alpha \nu_{A'}(x * y) \\
&\geq \alpha (1 - \min(\mu_{A'}(x), \mu_{A'}(y))) - \alpha \max(\nu_{A'}(x), \nu_{A'}(y)) \\
&\geq \alpha \min(x - \nu_{A'}(x), 1 - \nu_{A'}(y)) + (1 - \alpha) \min(\mu_{A'}(x), \mu_{A'}(y)) \\
&\geq \min \{ \alpha (1 - \nu_{A'}(x)) + (1 - \alpha) \mu_{A'}(x), \alpha (1 - \nu_{A'}(y)) + (1 - \alpha) \mu_{A'}(y) \} \\
&\geq \min \{ \mu_{A'}(x) + \alpha (1 - \mu_{A'}(x) - \nu_{A'}(x)), \mu_{A'}(y) + \alpha (1 - \mu_{A'}(y) - \nu_{A'}(y)) \} \\
&\geq \min \{ \mu_{F_{\alpha,\beta}}(A)(x), \mu_{F_{\alpha,\beta}}(A)(y) \} \\
\therefore \mu_{F_{\alpha,\beta}}(x * y) &\geq \min \{ \mu_{F_{\alpha,\beta}}(A)(x), \mu_{F_{\alpha,\beta}}(A)(y) \} \\
\text{Similarly we can prove that} \\
\nu_{F_{\alpha,\beta}}(x * y) &\leq \max \{ \nu_{F_{\alpha,\beta}}(A)(x), \nu_{F_{\alpha,\beta}}(A)(y) \}
\end{align*}
\]

Hence \( F_{\alpha,\beta}(A) \) is t-intuitionistic fuzzy subalgebra of X.

**Theorem 0.23** Cartesian product of two t-intuitionistic fuzzy subalgebra of X is again a t-intuitionistic fuzzy subalgebra of X \( \times X \).

**Proof.** Let \( A' = (\mu_{A'}, \nu_{A'}) \) and \( B' = (\mu_{B'}, \nu_{B'}) \) be two t-intuitionistic fuzzy subalgebra of BG-algebra X
Then their cartesion product \( A' \times B' = (\mu_{A' \times B'}, \nu_{A' \times B'}) \), where
\[
\begin{align*}
\mu_{A' \times B'}(x, y) &= \min \{ \mu_{A'}(x), \mu_{B'}(y) \} \\
\nu_{A' \times B'}(x, y) &= \max \{ \nu_{A'}(x), \nu_{B'}(y) \} \\
\forall x, y \in X.
\end{align*}
\]
Also
\[
\begin{align*}
\mu_{A'}(x * y) &\geq \min \{ \mu_{A'}(x), \mu_{A'}(y) \} & (4) \\
\nu_{A'}(x * y) &\leq \max \{ \nu_{A'}(x), \nu_{A'}(y) \} & (5)
\end{align*}
\]

\[
\begin{align*}
\mu_{A' \times B'}((x_1, y_1) \ast (x_2, y_2)) &= \mu_{A' \times B'}(x_1 * x_2, y_1 * y_2) \\
&= \min \{ \mu_{A'}(x_1 * x_2), \mu_{B'}(y_1 * y_2) \} \\
&\geq \min \{ \min \{ \mu_{A'}(x_1), \mu_{A'}(x_2) \}, \min \{ \mu_{B'}(y_1), \mu_{B'}(y_2) \} \} \\
&= \min \{ \min \{ \mu_{A'}(x_1), \mu_{B'}(y_1) \}, \min \{ \mu_{A'}(x_2), \mu_{B'}(y_2) \} \} \\
&= \min \{ \mu_{A' \times B'}((x_1, y_1), \mu_{A' \times B'}((x_2, y_2) \} \\
\Rightarrow \mu_{A' \times B'}((x_1, y_1) \ast (x_2, y_2)) &\geq \min \{ \mu_{A' \times B'}((x_1, y_1), \mu_{A' \times B'}((x_2, y_2) \}
\end{align*}
\]
Similarly we can show
\[
\begin{align*}
\nu_{A' \times B'}((x_1, y_1) \ast (x_2, y_2)) &\leq \max \{ \nu_{A' \times B'}((x_1, y_1), \nu_{A' \times B'}((x_2, y_2) \}
\end{align*}
\]
Corollary 0.24 If $A^t = < \mu_{A^t}, \nu_{A^t} >$ and $B^t = < \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy subalgebra of BG-algebra $X$. Then $\Box(A^t \times B^t) \triangleq (A^f \times B^f)$, $F_{\alpha, \beta}(A^f \times B^f)$ are also t-intuitionistic fuzzy subalgebra of $X \times X$.

Theorem 0.25 If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy normal subalgebra BG-algebra $X$, then $A$ is also t-intuitionistic fuzzy normal subalgebra of $X$.

Proof. Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy normal subalgebra BG-algebra $X$, therefore

\[(i)\mu_A((x \ast a) \ast (y \ast b)) \geq \min\{\mu_A(x \ast y), \mu_A(a \ast b)\}\]
\[(ii)\nu_A((x \ast a) \ast (y \ast b)) \leq \max\{\nu_A(x \ast y), \nu_A(a \ast b)\}, \forall x, y \in X.\]

Now, \[\mu_{A^t}((x \ast a) \ast (y \ast b)) = \min\{\mu_A((x \ast a) \ast (y \ast b)), t\} \geq \min\{\min\{\mu_A(x \ast y), \mu_A(a \ast b)\}, t\} = \min\{\min\{\mu_A(x \ast y), t\}, \min\{\mu_A(a \ast b), t\}\} = \min\{\mu_{A^t}(x \ast y), \mu_{A^t}(a \ast b)\}\]
\[\Rightarrow \mu_{A^t}((x \ast a) \ast (y \ast b)) \geq \min\{\mu_{A^t}(x \ast y), \mu_{A^t}(a \ast b)\}\]
Similarly we can show that \[\nu_{A^t}((x \ast a) \ast (y \ast b)) \leq \max\{\nu_{A^t}(x \ast y), \nu_{A^t}(a \ast b)\}\]

Hence $A$ is also t-intuitionistic fuzzy normal subalgebra BG-algebra $X$.

Remark 0.26 The converse of above Theorem is not true.

Theorem 0.27 If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set of BG-algebra $X$ and let $t < \min\{p, 1-q\}$, where $p = \min\{\mu_A(x) \mid x \in X\}$ and $q = \max\{\nu_A(x) \mid x \in X\}$ then $A$ is also t-intuitionistic fuzzy normal subalgebra BG-algebra $X$.

Proof. Same as Theorem 0.17.

Theorem 0.28 The intersection of two t-intuitionistic fuzzy normal subalgebra BG-algebra $X$ is also a t-intuitionistic fuzzy normal subalgebra of $X$.

Proof. Same as Theorem 0.19.

Theorem 0.29 If $A$ be IF normal subalgebra of BG-algebra $X$, then $\Box A, \triangle A$ and $F_{\alpha, \beta}(A)$ are also t-intuitionistic fuzzy normal subalgebra of $X$.

Proof. Same as Theorem 0.22.

Theorem 0.30 Cartesian product of two t-intuitionistic fuzzy normal subalgebra of $X$ is again a t-intuitionistic fuzzy normal subalgebra of $X \times X$.

Proof. Same as Theorem 0.23.

Corollary 0.31 If $A^t = < \mu_{A^t}, \nu_{A^t} >$ and $B^t = < \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy normal subalgebra of BG-algebra $X$. Then $\Box(A^t \times B^t), \triangle(A^f \times B^f), F_{\alpha, \beta}(A^f \times B^f)$ are also t-intuitionistic fuzzy normal subalgebra of $X \times X$.

Proof. Same as Corollary 0.24.
Homomorphism of t-intuitionistic fuzzy subalgebra \( BG \)-algebra

**Definition 0.32** Let \( X \) and \( Y \) be two \( BG \)-algebras, then a mapping \( f : X \rightarrow Y \) is said to be homomorphism if \( f(x * y) = f(x) * f(y), \forall x, y \in X \).

**Theorem 0.33** Let \( f : X \rightarrow Y \) be a homomorphism of \( BG \)-algebras, If \( A \) be a t-intuitionistic fuzzy subalgebra of \( Y \), then \( f^{-1}(A) \) is t-intuitionistic fuzzy subalgebra \( X \).

**Proof.** Let \( A \) be a t-intuitionistic fuzzy subalgebra of \( Y \). Let \( x, y \in X \) be any elements, then \( f^{-1}(A)(x * y) = (\mu_{f^{-1}(A)}(x * y), \nu_{f^{-1}(A)}(x * y)) \)

Now,\( \mu_{f^{-1}(A)}(x * y) \)
\[
= \mu_{A'}(x * y)
= \mu_{A'}[f(x) * f(y)]
\geq \min\{\mu_{A'}(f(x)), \mu_{A'}(f(y))\} \quad \text{[Since \( A \) is t-IF subalgebra of \( Y \)]}
= \min\{\mu_{f^{-1}(A)}(x), \mu_{f^{-1}(A)}(y)\}
\]
Therefore \( \mu_{f^{-1}(A)}(x * y) \geq \min\{\mu_{f^{-1}(A)}(x), \mu_{f^{-1}(A)}(y)\} \)

Similarly we can show that
\[
\nu_{f^{-1}(A)}(x * y) \leq \max\{\nu_{f^{-1}(A)}(x), \nu_{f^{-1}(A)}(y)\}
\]

Hence, \( f^{-1}(A') = (f^{-1}(A))^t \) is t-intuitionistic fuzzy subalgebra \( X \).

**Theorem 0.34** Let \( f : X \rightarrow Y \) be a homomorphism of \( BG \)-algebras, If \( A \) be a t-intuitionistic fuzzy normal subalgebra of \( Y \), then \( f^{-1}(A) \) is t-intuitionistic fuzzy normal subalgebra \( X \).

**Theorem 0.35** Let \( f : X \rightarrow Y \) be a onto homomorphism of \( BG \)-algebras, If \( A \) be t-intuitionistic fuzzy subalgebra \( X \). Then \( f(A) \) is t-intuitionistic fuzzy subalgebra of \( Y \).

**Proof.** Let \( y_1, y_2 \in Y \) Since \( f \) is onto, therefore there exists \( x_1, x_2 \in X \) such that \( f(x_1) = y_1, f(x_2) = y_2 \),
\[
f(A)(y_1 * y_2) = (\mu_{A'}(y_1 * y_2), \nu_{A'}(y_1 * y_2)) \quad \text{Now}
\]
\[
\nu_{A'}(y_1 * y_2) = \mu_A(t) \quad \text{where} \quad f(t) = y_1 * y_2 = f(x_1) * f(x_2) = f(x_1 * x_2)
\]
\[
\mu_{A'}(y_1 * y_2)
= \mu_{(f(A))}(y_1 * y_2)
= \min\{\mu_{f(A)}(y_1 * y_2), t\}
= \min\{\mu_{f(A)}(f(x_1) * f(x_2)), t\}
= \min\{\mu_{f(A)}(x_1 * x_2), t\}
= \mu_A(x_1 * x_2)
\geq \min\{\mu_{A'}(x_1), \mu_{A'}(x_2)\}, \quad \text{forall} \quad x_1, x_2 \in X \text{ such that } f(x_1) = y_1 \text{ and } f(x_1) = y_1
\]
\[
= \min\{\nu_{A'}(x_1)|f(x_1) = y_1, \nu_{A'}(x_2)|f(x_2) = y_2\}
= \min\{\nu_{f(A')}(y_1), \nu_{f(A')}(y_2)\}
\]
Therefore \( \mu_{A'}(y_1 * y_2) \geq \min\{\mu_{A'}(y_1), \mu_{A'}(y_2)\} \)

Similarly we can show that
\[
\nu_{A'}(y_1 * y_2) \leq \max\{\nu_{A'}(y_1), \nu_{A'}(y_2)\}
\]

Hence \( f(A) \) is t-intuitionistic fuzzy subalgebra of \( Y \).
Theorem 0.36 Let $f : X \to Y$ be a onto homomorphism of BG-algebras, If $A$ be t-intuitionistic fuzzy normal subalgebra of $X$, then $f(A)$ is t-intuitionistic fuzzy normal subalgebra of $Y$.

References


