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t- Intuitionistic Fuzzy Subalgebra of BG-Algebras

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Abstract. The aim of this paper is to introduced the notion of t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra of BG-algebras. We state and prove some theorems in t-intuitionistic fuzzy subalgebra and t-intuitionistic fuzzy normal subalgebra in BG-algebras. The homomorphic image and inverse image are investigated in both t-intuitionistic fuzzy subalgebra and normal subalgebras.

Introduction

In 1966, Imai and Iseki [6] introduced the two classes of abstract algebras, viz., BCK-algebras and BCI-algebras. It is known that the class of BCK-algebra is a proper subclass of the class of BCI-algebras. Neggers and Kim [8] introduced a new concept, called B-algebras, which are related to several classes of algebras such as BCI/BCK-algebras. Kim and Kim [7] introduced the notion of BG-algebra which is a generalization of B-algebra. The concept of intuitionistic fuzzy subset(IFS) was introduced by Atanassov [5] in 1983, which is a generalization of the notion of fuzzy sets. The concept of fuzzy subalgebras of BG-algebras was introduced by Ahn and Lee in [1]. The study of intuitionistic fuzzification of subalgebras and ideals of BG-algebras is done by Senapati et. al in [9]. The idea of t-intuitionistic fuzzy sets in fuzzy subgroups and fuzzy subrings is introduced by Sharma in[10, 11]. Here in this paper, we introduced the notion of t-intuitionistic fuzzy sets in fuzzy subalgebra and fuzzy normal subalgebras of BG-algebras and study their properties.

Preliminaries

Definition 0.1 *([1]) A BG-algebra is a non-empty set X with a constant $'0'$ and a binary operation $'*'$ satisfying the following axioms:*

(i) $x * x = 0$,

(ii) $x * 0 = x$,

(iii) $(x * y) * (0 * y) = x, \ \forall \ x, y \in X$.

For brevity, we also call X a BG-algebra. We can define a partial ordering $" \leq "$ on X by $x \leq y$ iff $x * y = 0$

Definition 0.2 *([1]) A non-empty subset S of a BG-algebra X is called a subalgebra of X if $x * y \in S$, for all $x, y \in S$.*

Definition 0.3 *Let X and Y be two non empty sets and $f : X \longrightarrow Y$ be a mapping. Let A and B be IFS's of X and Y respectively. Then the image of A under the mapf is denoted by f(A) and is defined by $f(A)(y) = (\mu_{f(A)}y, \nu_{f(A)}y)$, where* $\mu_{f(A)}(y) = \begin{cases} \vee\{\mu_A(x) : x \in f^{-1}(y)\} \\ 0 \quad \textit{otherwise} \end{cases}$ $\nu_{f(A)}(y) = \begin{cases} \wedge\{\nu_A(x) : x \in f^{-1}(y)\} \\ 1 \quad \textit{otherwise} \end{cases}$

also pre image of B under f is denoted by $f^{-1}(B)$ and is defined as $f^{-1}(B)(x) = (\mu_{f^{-1}(B)}, \nu_{f^{-1}(B)} = (\mu_B(f(x)), \nu_B(f(x))); \forall x \in X$
Remark Note that $\mu_A(x) \leq \mu_{f(A)}(f(x))$ and $\nu_A(x) \geq \nu_{f(A)}(f(x)) \quad \forall x \in X$ however equality hold when the map f is bijective.

Definition 0.4 *([2, 3]) An intuitionistic fuzzy set (IFS) A in a non empty set X is an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of non membership of the element x in the set A. For the sake of simplicity we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$. The function $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. is called the degree of uncertainty of $x \in A$. The class of IFSs on a universe X is denoted by $IFS(X)$.*

Definition 0.5 *([2, 3]) If $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ be any two IFS of a set X then*
$A \subseteq B$ iff for all $x \in X, \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$
$A = B$ iff for all $x \in X, \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$
$A \cap B = \{< x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x) > | x \in X\}$ where
$(\mu_A \cap \mu_B)(x) = min\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cup \nu_B)(x) = max\{\nu_A(x), \nu_B(x)\}$
$A \cup B = \{< x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x) > | x \in X\}$ where
$(\mu_A \cup \mu_B)(x) = max\{\mu_A(x), \mu_B(x)\}$ and $(\nu_A \cap \nu_B)(x) = min\{\nu_A(x), \nu_B(x)\}$

Definition 0.6 *([4]) For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and $\alpha \in [01]$, the operator $\Box : IFS(X) \to IFS(X), \Diamond : IFS(X) \to IFS(X), D_\alpha : IFS(X) \to IFS(X)$ are defined as*
(i) $\Box(A) = \{< x, \mu_A(x), 1 - \mu_A(x) > | x \in X\}$ is called necessity operator
(ii) $\Diamond(A) = \{< x, 1 - \nu_A(x), \nu_A(x) > | x \in X\}$ is called possibility operator
(iii) $D_\alpha(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + (1-\alpha)\pi_A(x) > | x \in X\}$ is called $\alpha-$ Model operator.
Clearly $\Box(A) \subseteq A \subseteq \Diamond(A)$ and the equality hold, when A is a fuzzy set also $D_0(A) = \Box(A)$ and $D_1(A) = \Diamond(A)$. Therefore the $\alpha-$ Model operator $D_\alpha(A)$ is an extension of necessity operator $\Box(A)$ and possibility operator $\Diamond(A)$.

Definition 0.7 *([4]) For any IFS $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ of X and for any $\alpha, \beta \in [01]$ such that $\alpha + \beta \leq 1$, the $(\alpha, \beta)-$ model operator $F_{\alpha,\beta} : IFS(X) \to IFS(X)$ is defined as $F_{\alpha,\beta}(A) = \{< x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x) > | x \in X\}$, where $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. Therefore we can write*
$F_{\alpha,\beta}(A)$ as $F_{\alpha,\beta}(A)(x) = (\mu_{F_{\alpha,\beta}(A)}(x), \nu_{F_{\alpha,\beta}(A)}(x))$
where $\mu_{F_{\alpha,\beta}}(x) = \mu_A(x) + \alpha\pi_A(x)$ and $\nu_{F_{\alpha,\beta}(A)}(x)) = \nu_A(x) + \beta\pi_A(x)$.
Clearly, $F_{0,1}(A) = \Box(A), F_{1,0}(A) = \Diamond(A)$ and $F_{\alpha,1-\alpha}(A) = D_\alpha(A)$

Definition 0.8 *([9]) An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ of a BG-algebra X is said to be an intuitionistic fuzzy subalgebra of X if*

$$(i)\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$$
$$(ii)\nu_A(x * y) \leq max\{\nu_A(x), \nu_A(y)\} \quad \forall x, y \in X.$$

Definition 0.9 *([7]) An IFS A of a BG-algebra X is said to be an IF normal subalgebra of X if*

(i) $\mu_A((x*a)*(y*b)) \geq min\{\mu_A(x*y), \mu_A(a*b)\}$,

(ii) $\nu_A((x*a)*(y*b)) \leq max\{\nu_A(x*y), \nu_A(a*b)\}$, $\forall\, x, y \in X$.

Definition 0.10 *([11]) Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of BG-algebra X. Let $t \in [0\ 1]$. then the intuitionistic fuzzy set A^t of X is called t-intuitionistic fuzzy subset(t-IFS) of X w.r.t A and is defined by $A^t = \{< x, \mu_{A^t}(x), \nu_{A^t}(x) > | x \in X\} =< \mu_{A^t}, \nu_{A^t} >$ where $\mu_{A^t}(x) = min\{\mu_A(x), t\}$ and $\nu_{A^t} = max\{\nu_A(x), 1-t\} \forall x \in X$*

Remark 0.11 *([11]) Let$A^t =< \mu_{A^t}, \nu_{A^t} >$ and $B^t =< \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy subsets of BG-algebra X, then*
$(A \cap B)^t = A^t \cap B^t$

Remark 0.12 *([11]) Let $f : X \to Y$ be a mapping. Let A and B are two IFS of X and Y respectively, then*
$(i) f^{-1}(B^t) = (f^{-1}(B))^t\ (ii) f(A^t) = (f(A))^t\ \ \forall t \in [0\ 1]$

Definition 0.13 *Let$A^t =< \mu_{A^t}, \nu_{A^t} >$ and $B^t =< \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy subsets of BG-algebra X Then their cartesian product $A^t \times B^t =< \mu_{A^t \times B^t}, \mu_{A^t \times B^t} >$ is defined by*
$\mu_{A^t \times B^t}(x, y) = min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$
$\nu_{A^t \times B^t}(x, y) = max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \quad \forall x, y \in X.$

t-Intuitionistic Fuzzy Subalgebra BG-algebra

Now onwards, let X denote a BG-algebra unless otherwise stated.

Definition 0.14 *Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set of BG-algebra X. Let $t \in [0\ 1]$ then A is called t-intuitionistic fuzzy subalgebra (t-IFSA) of X if A^t is IFSA of X i.e. if A^t satisfies following conditions:*

$$\begin{aligned} \mu_{A^t}(x*y) &\geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \\ \nu_{A^t}(x*y) &\leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \end{aligned}$$

Theorem 0.15 *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subalgebra BG-algebra X, then A is also t-intuitionistic fuzzy subalgebra of X.*

Proof. Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subalgebra BG-algebra X, therefore

$$\begin{aligned} &\mu_A(x*y) \geq min\{\mu_A(x), \mu_A(y)\} \\ &\nu_A(x*y) \leq max\{\nu_A(x), \nu_A(y)\}, \quad \forall x, y \in X. \end{aligned}$$

$$\begin{aligned} \text{Now,} \quad \mu_{A^t}(x*y) &= min\{\mu_A(x*y), t\} \\ &\geq min\{min\{\mu_A(x), \mu_A(y)\}, t\} \\ &= min\{min(\mu_A(x), t), min(\mu_A(y), t)\} \\ &= min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \\ \Rightarrow \mu_{A^t}(x*y) &\geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \end{aligned}$$

Similarly we can show

$$\nu_{A^t}(x*y) \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$$

Hence A is also t-intuitionistic fuzzy subalgebra BG-algebra X.

Remark 0.16 *The converse of above Theorem is not true.*

Example 1. Consider a BG-algebra $X = \{0, 1, 2\}$ with the following cayley table:

Table 1: Example of intuitionistic fuzzy BG-subalgebra.

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

The intuitionistic fuzzy subset $A = \{< x, \mu_A(x), \nu_A(x) > |x \in X\}$ given by $\mu_A(0) = 0.4$, $\mu_A(1) = 0.5, \mu_A(2) = 0.3$ and $\nu_A(0) = 0.5, \nu_A(1) = 0.4, \nu_A(2) = 0.6$. Since $\mu_A(0) = 0.4 \not\geq min\{\mu_A(1), \mu_A(1)\}$. Therefore A is not an intuitionistic fuzzy BG-subalgebra of X. Take $t = 0.2$. Then $\mu_A(x) > t$ for all $x \in X$ and also $\nu_A(x) < 1 - t$ for all $x \in X$.
Therefore $\mu_{A^t}(x * y) \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $\nu_{A^t}(x * y) \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$ for all $x \in X$. hold. Hence A is t-intuitionistic fuzzy subalgebra of X.

Theorem 0.17 *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set of BG-algebra X and let $t < min\{p, 1-q\}$, where $p = min\{\mu_A(x)|x \in X\}$ and $q = max\{\nu_A(x)|x \in X\}$ then A is also t-intuitionistic fuzzy subalgebra BG-algebra X.*

Proof. Since $t < min\{p, 1 - q\}$

$$\begin{aligned}
t &< min\{p, 1 - q\} \\
\Rightarrow\ & p > t \quad \text{and} \quad 1 - q > t \\
\Rightarrow\ & p > t \quad \text{and} \quad q < 1 - t \\
\Rightarrow\ & min\{\mu_A(x)|x \in X\} > t \quad \text{and} \quad max\{\nu_A(x)|x \in X\} < 1 - t \\
\Rightarrow\ & \mu_A(x) > t, \forall \quad x \in X \quad \text{and} \quad \nu_A(x) < 1 - t, \forall \quad x \in X
\end{aligned}$$

Therefore $\mu_{A^t}(x * y) \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\}$ and $\nu_{A^t}(x * y) \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\}$ for all $x \in X$ hold. Hence A is t-intuitionistic fuzzy subalgebra of X.

Theorem 0.18 *Any IF set of BG-algebra X can be realised as t-intuitionistic fuzzy subalgebra X.*

Proof. It follows from Theorem 0.17 and Theorem 0.15.

Theorem 0.19 *The intersection of two t-intuitionistic fuzzy subalgebra BG-algebra X is also a t-intuitionistic fuzzy subalgebra of X.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned}
\mu_{(A\cap B)^t}(x*y) &= min\{\mu_{(A\cap B)}(x*y), t\} \\
&\geq min\{min\{\mu_A(x*y), \mu_A(x*y)\}, t\} \\
&= min\{min(\mu_A(x*y), t), min(\mu_B(x*y), t)\} \\
&= min\{\mu_{A^t}(x*y), \mu_{B^t}(x*y)\} \\
&\geq min\{min\{\mu_{A^t}(x), \mu_{A^t}(y)\}, min\{\mu_{B^t}(x), \mu_{B^t}(y)\}\} \\
&= min\{min\{\mu_{A^t}(x), \mu_{B^t}(x)\}, min\{\mu_{A^t}(y), \mu_{B^t}(y)\}\} \\
&= min\{\mu_{(A\cap B)^t}(x), \mu_{(A\cap B)^t}(y)\} \\
\Rightarrow \mu_{(A\cap B)^t}(x*y) &\geq min\{\mu_{(A\cap B)^t}(x), \mu_{(A\cap B)^t}(y)\}
\end{aligned}$$

Similarly we can show that

$$\nu_{(A\cap B)^t}(x*y) \leq max\{\nu_{(A\cap B)^t}(x), \nu_{(A\cap B)^t}(y)\}$$

Theorem 0.20 *The intersection of any number of t-intuitionistic fuzzy subalgebra BG-algebra X is also a t-intuitionistic fuzzy subalgebra of X.*

Theorem 0.21 *For every t-intuitionistic fuzzy subalgebra A^t of X, the following properties hold*
(i) $\mu_{A^t}(0) \geq \mu_{A^t}(x)$
(ii) $\nu_{A^t}(0) \leq \nu_{A^t}(x), \ \forall x \in X.$

Proof. We have $\mu_{A^t}(0) = \mu_{A^t}(x*x) \geq min\{\mu_{A^t}(x), \mu_{A^t}(x)\} = \mu_{A^t}(x)$
and $\nu_{A^t}(0) = \nu_{A^t}(x*x) \leq max\{\nu_{A^t}(x), \nu_{A^t}(x)\} = \nu_{A^t}(x)$

Theorem 0.22 *If A be IF subalgebra of BG-algebra X, then $\Box A$, $\Diamond A$ and $F_{\alpha,\beta}(A)$ are also t-intuitionistic fuzzy subalgebra of X.*

Proof. Here A be IF subalgebra of BG-algebra X, By Theorem 0.15 A is also t-intuitionistic fuzzy subalgebra of X.

$$\mu_{A^t}(x*y) \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \tag{1}$$

$$\nu_{A^t}(x*y) \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \quad \forall x, y \in X. \tag{2}$$

Now $\Box A^t = \{< x, \mu_{A^t}(x), 1 - \mu_{A^t}(x) | x \in X\} = \{< x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) | x \in X\}$
$\Diamond A^t = \{< x, 1 - \nu_{A^t}(x), \mu_{A^t}(x) | x \in X\} = \{< x, \overline{\nu_{A^t}}(x), \mu_{A^t}(x) | x \in X\}$

Now

$$\begin{aligned}
\overline{\mu_{A^t}}(x*y) &= 1 - \mu_{A^t}(x*y) \\
&\leq 1 - min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \quad \text{By(1)} \\
&= max\{1 - \mu_{A^t}(x), 1 - \mu_{A^t}(y)\} \\
&= max\{\overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y)\}
\end{aligned}$$

$$\Rightarrow \overline{\mu_{A^t}}(x*y) \leq max\{\overline{\mu_{A^t}}(x), \overline{\mu_{A^t}}(y)\} \tag{3}$$

Hence by Eq^n (1) and (3) $\Rightarrow \Box A^t = \{< x, \mu_{A^t}(x), \overline{\mu_{A^t}}(x) | x \in X\}$ is t-intuitionistic fuzzy subalgebra of X.
Similarly we can show that

$\Diamond A^t = \{< x, \overline{\nu_{A^t}}(x), \mu_{A^t}(x) | x \in X\}$ is t-intuitionistic fuzzy subalgebra of X.
Again, we have $F_{\alpha,\beta}(A) =< \mu_{F_{\alpha,\beta}(A)}, \nu_{F_{\alpha,\beta}(A)} >$ let $x, y \in X$, then $F_{\alpha,\beta}(x * y) = (\mu_{F_{\alpha,\beta}(A)}(x * y), \nu_{F_{\alpha,\beta}(A)}(x * y))$ where $\mu_{F_{\alpha,\beta}(A)}(x * y) = \mu_A(x * y) + \alpha\pi_A(x * y)$ and $\nu_{F_{\alpha,\beta}(A)}(x * y) = \nu_A(x * y) + \beta\pi_A(x * y)$

$$\begin{aligned}
& \mu_{F_{\alpha,\beta}A^t}(x * y) \\
& = \mu_{A^t}(x * y) + \alpha\pi_{A^t}(x * y) \\
& = \mu_{A^t}(x * y) + \alpha(1 - \mu_{A^t}(x * y) - \nu_{A^t}(x * y)) \\
& \geq \alpha + (1 - \alpha)\mu_{A^t}(x * y) - \alpha\nu_{A^t}(x * y) \\
& \geq \alpha + (1 - \alpha)min(\mu_{A^t}(x), \mu_{A^t}(y)) - \alpha max(\nu_{A^t}(x), \nu_{A^t}(y)) \quad \text{By}(1) \\
& \geq \alpha\{1 - max(\nu_{A^t}(x), \nu_{A^t}(y))\} + (1 - \alpha)min(\mu_{A^t}(x), \mu_{A^t}(y)) \\
& \geq \alpha min(1 - \nu_{A^t}(x), 1 - \nu_{A^t}(y))\} + (1 - \alpha)min(\mu_{A^t}(x), \mu_{A^t}(y)) \\
& \geq min\{\alpha(1 - \nu_{A^t}(x)) + (1 - \alpha)\mu_{A^t}(x), \alpha(1 - \nu_{A^t}(y)) + (1 - \alpha)\mu_{A^t}(y)\} \\
& \geq min\{\mu_{A^t}(x) + \alpha(1 - \mu_{A^t}(x) - \nu_{A^t}(x)), \mu_{A^t}(y) + \alpha(1 - \mu_{A^t}(y) - \nu_{A^t}(y))\} \\
& \geq min\{\mu_{F_{\alpha,\beta}A^t}(x), \mu_{F_{\alpha,\beta}A^t}(y)\} \\
\therefore \quad & \mu_{F_{\alpha,\beta}A^t}(x * y) \geq min\{\mu_{F_{\alpha,\beta}A^t}(x), \mu_{F_{\alpha,\beta}A^t}(y)\} \\
& \text{Similarly we can prove that} \\
& \nu_{F_{\alpha,\beta}A^t}(x * y) \leq max\{\nu_{F_{\alpha,\beta}A^t}(x), \nu_{F_{\alpha,\beta}A^t}(y)\}
\end{aligned}$$

Hence $F_{\alpha,\beta}(A)$ is t-intuitionistic fuzzy subalgebra of X.

Theorem 0.23 *Cartesian product of two t-intuitionistic fuzzy subalgebra of X is again a t-intuitionistic fuzzy subalgebra of $X \times X$.*

Proof. Let$A^t =< \mu_{A^t}, \nu_{A^t} >$ and $B^t =< \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy subalgebra of BG-algebra X
Then their cartesian product $A^t \times B^t =< \mu_{A^t \times B^t}, \mu_{A^t \times B^t} >$, where
$\mu_{A^t \times B^t}(x, y) = min\{\mu_{A^t}(x), \mu_{B^t}(y)\}$
$\nu_{A^t \times B^t}(x, y) = max\{\nu_{A^t}(x), \nu_{B^t}(y)\} \quad \forall x, y \in X.$
Also

$$\mu_{A^t}(x * y) \quad \geq min\{\mu_{A^t}(x), \mu_{A^t}(y)\} \tag{4}$$
$$\nu_{A^t}(x * y) \quad \leq max\{\nu_{A^t}(x), \nu_{A^t}(y)\} \quad \forall x, y \in X. \tag{5}$$

$$\begin{aligned}
\mu_{A^t \times B^t}((x_1, y_1) * (x_2, y_2)) &= \mu_{A^t \times B^t}(x_1 * x_2, y_1 * y_2) \\
&= min\{\mu_{A^t}(x_1 * x_2), \mu_{B^t}(y_1 * y_2)\} \\
&\geq min\{min\{\mu_{A^t}(x_1), \mu_{A^t}(x_2)\}, min\{\mu_{B^t}(y_1), \mu_{B^t}(y_2)\}\} \\
&= min\{min\{\mu_{A^t}(x_1), \mu_{B^t}(y_1)\}, min\{\mu_{A^t}(x_2), \mu_{B^t}(y_2)\}\} \\
&= min\{\mu_{A^t \times B^t}((x_1, y_1), \mu_{A^t \times B^t}((x_2, y_2)\} \\
\Rightarrow \mu_{A^t \times B^t}((x_1, y_1) * (x_2, y_2)) &\geq min\{\mu_{A^t \times B^t}((x_1, y_1), \mu_{A^t \times B^t}((x_2, y_2)\} \\
\text{Similarly we can show} & \\
\nu_{A^t \times B^t}((x_1, y_1) * (x_2, y_2)) &\leq max\{\nu_{A^t \times B^t}((x_1, y_1), \nu_{A^t \times B^t}((x_2, y_2)\}
\end{aligned}$$

Corollary 0.24 *If $A^t =< \mu_{A^t}, \nu_{A^t} >$ and $B^t =< \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy subalgebra of BG-algebra X. Then $\Box(A^t \times B^t)$ $\Diamond(A^t \times B^t)$, $F_{\alpha,\beta}(A^t \times B^t)$ are also t-intuitionistic fuzzy subalgebra of $X \times X$.*

Theorem 0.25 *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy normal subalgebra BG-algebra X, then A is also t-intuitionistic fuzzy normal subalgebra of X.*

Proof. Since $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy normal subalgebra BG-algebra X, therefore

$$(i)\mu_A((x*a)*(y*b)) \geq min\{\mu_A(x*y), \mu_A(a*b)\}$$
$$(ii)\nu_A((x*a)*(y*b)) \leq max\{\nu_A(x*y), \nu_A(a*b)\},\ \forall\ x, y \in X.$$

$$\begin{aligned}
\text{Now,}\quad \mu_{A^t}((x*a)*(y*b)) &= min\{\mu_A((x*a)*(y*b)), t\} \\
&\geq min\{min\{\mu_A(x*y), \mu_A(a*b)\}, t\} \\
&= min\{min(\mu_A(x*y), t), min(\mu_A(a*b), t)\} \\
&= min\{\mu_{A^t}(x*y), \mu_{A^t}(a*b)\} \\
\Rightarrow \mu_{A^t}((x*a)*(y*b)) &\geq min\{\mu_{A^t}(x*y), \mu_{A^t}(a*b)\} \\
\text{Similarly we can show that} & \\
\nu_{A^t}((x*a)*(y*b)) &\leq max\{\nu_{A^t}(x*y), \nu_{A^t}(a*b)\}
\end{aligned}$$

Hence A is also t-intuitionistic fuzzy normal subalgebra BG-algebra X.

Remark 0.26 *The converse of above Theorem is not true.*

Theorem 0.27 *If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy set of BG-algebra X and let $t < min\{p, 1-q\}$, where $p = min\{\mu_A(x)|x \in X\}$ and $q = max\{\nu_A(x)|x \in X\}$ then A is also t-intuitionistic fuzzy normal subalgebra BG-algebra X.*

Proof. Same as Theorem 0.17.

Theorem 0.28 *The intersection of two t-intuitionistic fuzzy normal subalgebra BG-algebra X is also a t-intuitionistic fuzzy normal subalgebra of X.*

Proof. Same as Theorem 0.19.

Theorem 0.29 *If A be IF normal subalgebra of BG-algebra X,then $\Box A, \Diamond A$ and $F_{\alpha,\beta}(A)$are also t-intuitionistic fuzzy normal subalgebra of X.*

Proof. Same as Theorem 0.22.

Theorem 0.30 *Cartesian product of two t-intuitionistic fuzzy normal subalgebra of X is again a t-intuitionistic fuzzy normal subalgebra of $X \times X$.*

Proof. Same as Theorem 0.23.

Corollary 0.31 *If $A^t =< \mu_{A^t}, \nu_{A^t} >$ and $B^t =< \mu_{B^t}, \nu_{B^t} >$ be two t-intuitionistic fuzzy normal subalgebra of BG-algebra X. Then $\Box(A^t \times B^t)$, $\Diamond(A^t \times B^t)$, $F_{\alpha,\beta}(A^t \times B^t)$ are also t-intuitionistic fuzzy normal subalgebra of $X \times X$.*

Proof. Same as Corollary 0.24.

Homomorphism of t-intuitionistic fuzzy subalgebra BG-algebra

Definition 0.32 *Let X and Y be two BG-algebras, then a mapping $f : X \longrightarrow Y$ is said to be homomorphism if $f(x * y) = f(x) * f(y),\ \forall x, y \in X$.*

Theorem 0.33 *Let $f : X \longrightarrow Y$ be a homomorphism of BG-algebras, If A be a t-intuitionistic fuzzy subalgebra of Y, then $f^{-1}(A))$ is t-intuitionistic fuzzy subalgebra X.*

Proof. A be a t-intuitionistic fuzzy subalgebra of Y. Let $x, y \in X$ be any elements, then $f^{-1}(A^t)(x * y) = (\mu_{f^{-1}(A^t)}(x * y), \nu_{f^{-1}(A^t)}(x * y))$

Now,$\mu_{f^{-1}(A^t)}(x * y)$
$= \mu_{A^t} f(x * y)$
$= \mu_{A^t}[f(x) * f(y)]$
$\geq min\{\mu_{A^t}(f(x)), \mu_{A^t}(f(y))\}$ [Since A is t-IF subalgebra of Y]
$= min\{\mu_{f^{-1}A^t}(x), \mu_{f^{-1}A^t}(y)\}$
Therefore $\mu_{f^{-1}A^t}(x * y) \geq min\{\mu_{f^{-1}A^t}(x), \mu_{f^{-1}A^t}(y)\}$
Similarly we can show that
$\nu_{f^{-1}(A^t)}(x * y) \leq max\{\nu_{f^{-1}A^t}(x), \nu_{f^{-1}A^t}(y)\}$

Hence, $f^{-1}(A^t) = (f^{-1}(A))^t$ is t-intuitionistic fuzzy subalgebra X.

Theorem 0.34 *Let $f : X \longrightarrow Y$ be a homomorphism of BG-algebras, If A be a t-intuitionistic fuzzy normal subalgebra of Y, then $f^{-1}(A)$ is t-intuitionistic fuzzy normal subalgebra X.*

Theorem 0.35 *Let $f : X \longrightarrow Y$ be a onto homomorphism of BG-algebras, If A be t-intuitionistic fuzzy subalgebra X. Then $f(A)$ is t-intuitionistic fuzzy subalgebra of Y.*

Proof. Let $y_1, y_2 \in Y$ Since f is onto, therefore there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1, f(x_2) = y_2$,
$f(A)(y_1 * y_2) = (\mu_{f(A)}(y_1 * y_2), \nu_{f(A)}(y_1 * y_2))$, Now
$\mu_{f(A)}(y_1 * y_2) = \mu_A(t)$ where $f(t) = y_1 * y_2 = f(x_1) * f(x_2) = f(x_1 * x_2)$

$\mu_{f(A^t)}(y_1 * y_2)$
$= \mu_{(f(A))^t}(y_1 * y_2)$
$= min\{\mu_{f(A)}(y_1 * y_2), t\}$
$= min\{\mu_{f(A)}(f(x_1) * f(x_2)), t\}$
$= min\{\mu_{f(A)} f(x_1 * x_2), t\}$
$= min\{\mu_A(x_1 * x_2), t\}$
$= \mu_{A^t}(x_1 * x_2)$
$\geq min\{\mu_{A^t}(x_1), \mu_{A^t}(x_2)\}$, forall $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_1) = y_1$
$= min\{\vee\{\mu_{A^t}(x_1)|f(x_1) = y_1\}, \vee\{\mu_{A^t}(x_2)|f(x_2) = y_2\}\}$
$= min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$
Therefore $\mu_{f(A^t)}(y_1 * y_2) \geq min\{\mu_{f(A^t)}(y_1), \mu_{f(A^t)}(y_2)\}$
Similarly we can show that
$\nu_{f(A^t)}(y_1 * y_2) \leq max\{\nu_{f(A^t)}(y_1), \nu_{f(A^t)}(y_2)\}$

Hence $f(A)$ is t-intuitionistic fuzzy subalgebra of Y.

Theorem 0.36 *Let $f : X \longrightarrow Y$ be a onto homomorphism of BG-algebras, If A be t-intuitionistic fuzzy normal subalgebra X, then $f(A)$ is t-intuitionistic fuzzy normal subalgebra of Y.*

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