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A proof of the Schinzel conjecture

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Abstract
In this paper, we deal with the Schinzel conjecture.

The proof
The Schinzel conjecture states that

$$\forall n > v, v \in N, \exists x, y, z' \in N \mid \frac{v}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z'}$$

LEMMA 1

Let

$$u = x + y \tag{1}$$

$$\frac{1}{z} = \frac{1}{x} + \frac{1}{y} \tag{2}$$

Let us build the sequences. If we pose

$$x_1 = x$$

$$y_1 = y$$

And $\forall x_1, y_1, \text{integers}, \exists z_1$ verifying

$$\frac{1}{z_1} = \frac{1}{x_1} + \frac{1}{y_1}$$

And

$$z_1 = \frac{xy}{x+y} = z$$

So

$$(x_1 + y_1)z_1 = x_1 y_1$$

And

$$x_1(y_1 - z_1) = x_1 y_1$$

We pose

$$y_2 = y_1 - z_1 = \frac{z_1 y_1}{x_1}$$

Also

$$y_1(x_1 - z_1) = \frac{z_1 x_1}{y_1}$$

We pose

$$x_2 = x_1 - z_1 = \frac{z_1 x_1}{y_1}$$

And

$$x_2 y_2 = z_1^{\ 2}$$

Which means that

$$x_1 = x_2 + z_1 = x_2 + \sqrt{x_2 y_2}$$

$$y_1 = y_2 + z_1 = y_2 + \sqrt{x_2 y_2}$$

$$u_1 = u = (x_1 + y_1) = (\sqrt{x_2} + \sqrt{y_2})^2 > x_2 + y_2 > 0$$

So

$$x_1 = \sqrt{x_2}(\sqrt{x_2} + \sqrt{y_2}) > x_2 > 0$$

$$y_1 = \sqrt{y_2}(\sqrt{x_2} + \sqrt{y_2}) > y_2 > 0$$

$$z_1 = \frac{x_1 y_1}{x_1 + y_1} = \sqrt{x_2 y_2} > z_2 = \frac{x_2 y_2}{x_2 + y_2} > 0$$

Because $\forall x_2, y_2, \exists z_2$ verifying

$$\frac{1}{z_2} = \frac{1}{x_2} + \frac{1}{y_2}$$

The process is available until infinity. For

$$u_i = x_i + y_i = (\sqrt{x_{i+1}} + \sqrt{y_{i+1}})^2 > x_{i+1} + y_{i+1} > 0$$

$$x_i = \sqrt{x_{i+1}}(\sqrt{x_{i+1}} + \sqrt{y_{i+1}}) > x_{i+1} > 0$$

$$y_i = \sqrt{y_{i+1}}(\sqrt{x_{i+1}} + \sqrt{y_{i+1}}) > y_{i+1} > 0$$

$$z_i = \frac{x_i y_i}{x_i + y_i} = \sqrt{x_{i+1} y_{i+1}} > z_{i+1} = \frac{x_{i+1} y_{i+1}}{x_{i+1} + y_{i+1}} > 0$$

And, of course

$$\frac{1}{z_{i+1}} = \frac{1}{x_{i+1}} + \frac{1}{y_{i+1}}$$

We have built the sequences.

LEMMA 2

x_i, y_i have an expression

$$x_i = x^{2^{i-1}} \prod_{j=0}^{j=i-2} (x^{2^j} + y^{2^j})^{-1} \qquad \text{(H)}$$

$$y_i = y^{2^{i-1}} \prod_{j=0}^{j=i-2} (x^{2^j} + y^{2^j})^{-1} \qquad \text{(H')}$$

Proof of lemma 2

By traditional induction, for i=2

$$x = \sqrt{x_2}(\sqrt{x_2} + \sqrt{y_2}) = \sqrt{x_2}(x+y)^{\frac{1}{2}}$$

$$x_2 = \frac{x^2}{x+y}$$

Also

$$y = \sqrt{y_2}(\sqrt{x_2} + \sqrt{y_2}) = \sqrt{y_2}(x+y)^{\frac{1}{2}}$$

$$y_2 = \frac{y^2}{x+y}$$

We suppose (H) and (H') true for i, then

$$x_i = \sqrt{x_{i+1}}(\sqrt{x_{i+1}} + \sqrt{y_{i+1}}) = \sqrt{x_{i+1}}(x_i + y_i)^{\frac{1}{2}}$$

$$x_{i+1} = \frac{x_i^2}{x_i + y_i} = x^{2^i} \prod_{j=0}^{j=i-2} (x^{2^j} + y^{2^j})^{-2} (x^{2^{i-1}} + y^{2^{i-1}})^{-1} \prod_{j=0}^{j=i-2} (x^{2^j} + y^{2^j}) = x^{2^i} \prod_{j=0}^{j=i-1} (x^{2^j} + y^{2^j})^{-1}$$

Also

$$y_i = \sqrt{y_{i+1}}(\sqrt{x_{i+1}} + \sqrt{y_{i+1}}) = \sqrt{y_{i+1}}(x_i + y_i)^{\frac{1}{2}}$$

$$y_{i+1} = \frac{y_i^2}{x_i + y_i} = y^{2^i} \prod_{j=0}^{j=i-2} (x^{2^j} + y^{2^j})^{-2} (x^{2^{i-1}} + y^{2^{i-1}})^{-1} \prod_{j=0}^{j=i-2} (x^{2^j} + y^{2^j}) = y^{2^i} \prod_{j=0}^{j=i-1} (x^{2^j} + y^{2^j})^{-1}$$

It is proved, but $\forall x, y$

$$\prod_{j=0}^{j=i-2} (x^{2^j} + y^{2^j}) = \frac{x^{2^{i-1}} - y^{2^{i-1}}}{x - y}$$

Then, for $x \neq y$

$$x_i = \frac{x^{2^{i-1}}}{x^{2^{i-1}} - y^{2^{i-1}}}(x-y)$$

$$y_i = \frac{y^{2^{i-1}}}{x^{2^{i-1}} - y^{2^{i-1}}}(x-y)$$

LEMMA 3 and Proof.

Fermat equation has the constant

$$x_i - y_i = x - y$$

But, with the same building, we have the sequences

$$\frac{v}{n} = \frac{1}{v/n} = \frac{1}{z} + \frac{1}{z'} = \frac{1}{z''}$$

$$z_i = \frac{z^{2^{i-1}}}{z^{2^{i-1}} - z'^{2^{i-1}}}(z-z')$$

$$z'_i = \frac{z'^{2^{i-1}}}{z^{2^{i-1}} - z'^{2^{i-1}}}(z-z')$$

$$z_i - z'_i = z - z'$$

Let

$$w_i = \frac{z^{a2^{i-1}}}{z^{a2^{i-1}} - z'^{a2^{i-1}}}(z-z')$$

$$p_i = \frac{z'^{a2^{i-1}}}{z^{a2^{i-1}} - z'^{a2^{i-1}}}(z-z')$$

$$w'_i = \frac{z^{b2^{i-1}}}{z^{b2^{i-1}} - z'^{b2^{i-1}}}(z-z')$$

$$p'_i = \frac{z'^{b2^{i-1}}}{z^{b2^{i-1}} - z'^{b2^{i-1}}}(z-z')$$

$$w''_i = \frac{z^{(b-a)2^{i-1}}}{z^{(b-a)2^{i-1}} - z'^{(b-a)2^{i-1}}}(z-z')$$

$$p''_i = \frac{z'^{(b-a)2^{i-1}}}{z^{(b-a)2^{i-1}} - z'^{(b-a)2^{i-1}}}(z-z')$$

$$w_i - p_i = z - z' = w'_i - p'_i = z_i - z'_i = w''_i - p''_i$$

But

$$\frac{w_i}{p_i} = \frac{z^{a2^{t-1}}}{z'^{a2^{t-1}}}$$

$$\frac{w'_i}{p'_i} = \frac{z^{b2^{t-1}}}{z'^{b2^{t-1}}} = \frac{z^{a2^{t-1}}}{z'^{a2^{t-1}}} \frac{z^{(b-a)2^{t-1}}}{z'^{(b-a)2^{t-1}}} = \frac{w_i w''_i}{p_i p''_i}$$

$$\Rightarrow \frac{w'_i p_i - p'_i w_i}{p'_i w_i} = \frac{(w'_i - p'_i)p_i + p'_i(p_i - w_i)}{p'_i w_i} = \frac{(z - z')(p'_i - p_i)}{p'_i w_i}$$

$$= \frac{w''_i - p''_i}{w''_i} = \frac{z - z'}{w''_i}$$

$$(p'_i - p_i)w''_i \neq p'_i w_i \Rightarrow z - z' = 0$$

It means that

$$\frac{v}{n} = \frac{2}{z} = \frac{2}{x} + \frac{2}{y} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z'}$$

And the conjecture is not true.

Conclusion :

By series of formulas and sequances, we have proved that the Schinzel conjecture is false.